# Variable Payout Annuities with Downside Protection: How to Replace the Lost Longevity Insurance in DC Plans

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#### Summary Abstract

Motivated by the rapid decline of traditional defined benefit (DB) pension plans and their implicit longevity insurance, in this report we quantify the value of having access to variable payout annuities with downside protection inside an individually-controlled defined contribution (DC) plan. We deliberately use the term variable payout annuity (VPA) to emphasize an instrument whose sole purpose is to generate retirement income, i.e. to eventually annuitize and convert into an immediate variable annuity (IVA).

VPAs are an important component of a well diversified portfolio at all stages of the human life cycle, especially for those who lack a traditional DB pension. In addition -- for those who are concerned about the retirement income volatility -having some form of downside protection during the payout stage is "worth" paying for and hence improves the risk & return properties of a payout annuity.

Our simulations indicate that -- although over long periods of time a balanced portfolio of equities and bonds are likely to appreciate -- the value of an implicit annuitization put option (APO) on such a portfolio can be substantial.

In addition to developing some metrics for explicitly quantifying this value, we provide a variety of numerical examples and case studies under a particular form of downside protection.

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#### 1.) Introduction and Motivation: The Slow Death of Longevity Insurance

During the year 2002 a historically unprecedented pension experiment took place in the State of Florida. Every one of the State's more than 500,000 public employees – in addition to every new employee joining the State's payroll -- was given the option of converting their traditional Defined Benefit (DB) pension plan into an individually managed Defined Contribution (DC) plan. The DC investment plan was very similar to a corporate-style 401(k) plan, under which the employee has full control over asset allocation and investment decisions. The new Public Employee Optional Retirement Program (PEORP), as it has been called, was the focus of intense scrutiny by local and national media. This is because it was the largest such pension conversion in the history of the U.S. and was viewed by many observers as a potential laboratory for Social Security reform. It is estimated that over 50% of new employees of the State have decided to forgo the traditional DB pension and instead enroll in the DC investment plan.

This large-scale transition from DB to DC is not isolated to the State of Florida alone. A number of other States -- including a failed attempt by California Governor Arnold Schwarzenegger -- have proposed converting their State's public employee DB plan into either a mandatory or optional DC plan. The impetus for this massive shift can be attributed to a wide variety of factors, but is primarily due to the actuarial funding crisis that has been brewing for many years. The economic cost of funding and maintaining DB pensions has reached unprecedented levels, driven by low interest rates, poor performance of the equity markets and the uncertainty of increasing life spans. A recent cover story in *Business Week* in June of 2005 brought this crisis to national prominence.

Private sector corporate pension plans have not been immune to this trend either. In aggregate, DB pension plans in the U.S. have a collective funding deficit in the hundreds of billions of dollars, depending on which assumptions are used to discount liabilities. They, too, have suffered from the same increasing longevity patterns, declining interest rates, poor equity returns and a cumbersome regulatory environment. It is no surprise, then, that according to the U.S. Department of Labor, the number of private sector DB plans in the U.S. has fallen from 112,208 in the year 1980 to 29,512 in the year 2003. Likewise, the number of private sector employees covered by a DB plan has fallen from 30.1 million in 1980 to 22.6 million in early 2000. More interestingly, the percentage of private sector employees covered

by a DB plan has fallen from 28% in 1980 to 7% in early 2000. In sum, Defined Benefit pension plans are dying.

For the most part, the vacuum created by the demise of DB pension coverage has been taken-up by DC style accounts such as 401(k), 403(b), 457, etc. plans. According to LIMRA International, the percent of U.S. individuals covered by a DC plan increased from 17% in 1980 to 58% in 1999. These DC plans differ from their DB cousins by providing control over asset allocation, mobility, flexibility and greater transparency. Indeed, many consumer advocates and free-market economists have argued that this trend – from DB to DC – is a movement in the right direction as the U.S. labor force transitions from traditional lifelong manufacturing jobs to service-type employment.

This report remains neutral as to whether DC plans are better (or worse) for the general workforce and we refrain from this contentious debate as it relates to transaction costs and consumer behavior. Moreover, we concede up-front that a DC-style plan offers a number of benefits – especially for younger, professional and mobile employees -- that are simply unavailable under a DB arrangement.

However, there is one aspect of DC plans – as currently embodied in most 401(k), 403(b) and similar structures -- that places them at a disadvantage relative to traditional DB plans. We believe that this problem can be remedied by expanding the set of menu options available within DC plans together with appropriate education for plan participants and plan sponsors.

The Achilles Heel of DC plans is that regardless of how much money plan participants manage to accumulate in their accounts and despite how successful they might be in managing their financial affairs, in the words of pension economists, they have lost their *longevity insurance*. That is, they have lost the guarantee of lifetime income that is an integral part of a DB plan. Indeed, a retiree with a DC-style plan who decides to either roll-over into an IRA or take a lump-sum settlement and create systematic withdrawal plans to finance their retirement consumption, loses the longevity insurance against the cost of living far longer than they had anticipated.

This, in fact, is the core message of this report. No matter how large the menu of mutual funds, wrap accounts, ETFs or low-cost index funds the plan sponsors offer participants, most DC-style plans do not offer financial products that contain – or have the ability to create -- longevity insurance. According to a recent study by LIMRA International,

fewer than three in ten 401(k) plan sponsors offer any type of payout annuity as the normal form of distribution. The number of plan participants who voluntarily annuitize – that is convert part or all of their nest egg into a lifetime income stream – is even smaller.

There are clearly two levels of decision making that must take place when it comes to retirement income planning. The first tier revolves around whether to finance withdrawals and consumption needs from a systematic withdrawal plan (SWiP) – where the retiree retains longevity risk – or whether to outsource or transfer this risk to an insurance company who is better able to pool this risk amongst a large population. This is known as the annuitization option. Of course, these two options are not mutually exclusive and a recommended possibility would be to diversify across a SWiP program and an income annuity. It is our belief that this first tier decision should not be delayed until retirement and careful attention must be devoted to longevity risk well before retirement. The Milevsky (2005) article, recently published in the North American Actuarial Journal, further elaborates on and argues this position. Once this first tier decision has been made, and a commitment has be made to fund some portion of retirement income using an annuity instrument, a second tier decision revolves around the optimal timing of annuitization and the type of annuity to purchase. For example, some variable payout annuities provide fluctuating income with no guaranteed base, while others protect a baseline level of income regardless of market performance. Some payout annuities protect the retiree against inflation risk, while others make payments in nominal terms. Finally, the second tier decision involves another dimension, which concerns guarantee periods and survivorship benefits.

We believe that DC plan sponsors should give serious consideration to providing a diversified menu of *asset classes and product classes* to help participants prepare for a retirement in which they must generate lifetime income. In fact, even those plan sponsors that do offer a life annuity as the primary form of distribution, only offer a fixed (nominal) payment product that is unlikely to keep up with the retirees' cost of living.

Longevity insurance is a very peculiar and odd-sounding form of insurance and it takes some thinking to understand how this insurance is embedded within DB pension plans, payout annuities, and other insurance products. Most consumers understand the mechanics of life, car, home or health insurance. A premium is paid upfront to protect one's family and possessions against a catastrophic financial event, such as the death or disability of the primary breadwinner. These commodity-type insurance policies have few financial characteristics and one rarely thinks of them as providing an investment rate of return. Traditional insurance policies <u>hedge</u> the family against catastrophic financial events.

Yet, the same line of thought can also be extended to insurance protection towards the end of the human life cycle, when the risks a retiree faces are of the exact opposite magnitude. As one ages and transitions into retirement, the value of human capital (future labor income) dwindles and all one has to support themselves during their extended retirement years is the financial capital they have amassed during the working years. At that point in their life, their future consumption *liabilities* are uncertain and unpredictable. The retiree may be fortunate enough and live a very long life well into their 90s, or they may be unlucky and barely reach a typical retiree's life-expectancy of 80. This uncertainty can be hedged or diversified away by being a member of a DB pension plan or by voluntarily purchasing a life annuity, which implicitly provides a higher rate of return, the longer one lives. So, although their liabilities might increase beyond what was expected if they reach their 90s, so too will the investment return from a life annuity, or what we call longevity insurance.

This is why a number of public commentators -- including the primary author of this report -- have argued that variable payout annuities (VPAs), which are converted into immediate variable annuities (IVA), should form the backbone of one's retirement income portfolio. In this report we echo this position and take it one step further.

We acknowledge that one of the primary concerns with linking retirement income to the performance of an investment portfolio, is the volatility. Therefore, in addition to serving as an advocacy piece for VPAs, the technical objective of this report is to examine the benefits and costs of having access to a downside-protected, or guaranteed VPA.

From an economic point of view, a guaranteed VPA (GVPA) is a savings & insurance contract that provides an option to annuitize the account at some pre-specified price, with an additional guarantee that the lifetime income will <u>not</u> fall below some baseline level. The GVPA becomes a GIVA, which can be thought of as an IVA with a put option. We will describe this in greater detail, later in the analysis.

The remainder of this report is organized as follows. Section 2 reviews the mechanics of the pure longevity insurance, which is embedded within a variable (or fixed) payout annuity. It helps to translate the language of insurance to the language of investments. Section 3 discusses the general structure of variable payout annuities that provide downside

protection and Section 4 goes on to report on extensive simulation results for the relative value of having this downside protection for one particular product design. Section 5 concludes the report with some final thoughts and a Technical Appendix contains the bulk of the mathematical detail. All simulation results are displayed at the end of the document.

# 2.) Understanding the Benefits of an Immediate Annuity (without having a Degree in Actuarial Science)

In its most general form, purchasing an immediate annuity (whether joint or single, for life or for a fixed-term) involves paying a non-refundable lump sum to an insurance company in exchange for a guaranteed, constant monthly or quarterly income. With some products, the income ends after a pre-determined or fixed period of time; these are called fixed-term (or period certain) annuities. With life annuity products, the income ends at death.

Obviously, a retiree cannot outlive the income from a life annuity; this is one of the product's strong selling points. No matter how long you live, how markets perform, or what happens to interest rates or the economy as a whole, you will always get a monthly cheque. Annuities, in other words, are a type of longevity insurance.

Insurance companies can provide this lifelong benefit by (a) pooling a large enough group of annuitants and (b) making a very careful and conservative assumption about the rate of return earned on its assets. The pooling of annuitants means that individuals who do not reach their life expectancy, as calculated by actuarial mortality tables, will end up subsidizing those who exceed it.

A simple example can help convert longevity insurance embedded within annuities and DB pension plans into the language of investments. Suppose, according to population mortality tables, there is a 20 percent chance that a 95-year-old female will die during the next year, before she reaches her 96<sup>th</sup> birthday. If 1,000 such females enter into a one year term annuity (a.k.a. tontine) agreement by investing \$100 each in a pool yielding 5 percent, the funds will grow to \$105,000 by the end of the year. Of the initial 1,000 females, 800 are expected to survive, with a rather small variance around the expected value, leaving an average of \$105,000/800 = \$131.25 per survivor. This leads to a total return of 31.25% and quite obviously far exceeds the interest rate (or investment return) of 5% used to "store" the funds, because the annuitants have ceded control of assets in the event of death. The powerful algebra of longevity credits can be stated symbolically as follows: if r denotes the effective interest rate per year and if p is the probability of survival per year, then the return for the survivors from the one-year annuity is expected to be (1 + r)/p - 1 > r. The expectation will become reality as long as the group of annuitants participating in this risk-mitigating scheme is large enough. The gap between the one-year returns to the survivors and the interest rate are the so-called mortality credits. Table #1a illustrates some numerical values for these credits at different ages, using a unisex annuitant mortality table.

Table 1a								
The Investment Be	nefits from a One-Year Term Annuity							
Age	Mortality Credits (b.p.)							
55	35							
60	52							
65	83							
70	138							
75	237							
80	414							
85	725							
90	1256							
95	2004							
Assuming 40	/60 male/female split							
for Annuity 2	000 Table under 6% interest							

To put these numbers in perspective, a (unisex) 85 year-old that decides <u>not</u> to buy the *one-year term annuity* and instead *take his or her chances* by investing in traditional (non-mortality contingent) asset classes, would have to earn 725 basis points (which is 7.25%) above the risk-free rate of 6% during the next year, in order to be as well-off as someone who decided to buy the one-year term annuity at age 85. Think of this (6% + 7.25% =) 13.25% number as a hurdle-rate that must be earned by the self-annuitizer to keep up with the "annuitizer." At age 90 this hurdle rate increases to 18.56%, which becomes virtually unachievable using any conventional investment products. Of course, different interest rates and mortality tables will lead to different numerical results, but the order of magnitude is always the same. *At advanced ages nothing beats the implied yield from an income annuity*.

As many pension actuaries understand and appreciate, the risk-sharing principle of "tontine insurance" is in fact the concept underlying all immediate annuities, and all DB pension plans for that matter. In practice, however, the risk-sharing agreement is made over a series of years, as opposed to just one year. We will elaborate on the distinction in a moment. Consider what we described as *term* longevity insurance versus *whole-life* longevity insurance. The mechanics remain the same, and the survivors derive a higher return – which is then amortized over one's life -- compared to placing the funds in a conventional (non-mortality-contingent) asset.

More importantly, while the above example assumes that the interest rate *r* is fixed, in theory, the exact same principle applies with a variable investment return as well. In fact, the *ex post* returns might be even higher. For example, the 1,000 females who are 95 years-old, can invest their \$100 in a balanced mutual fund that earns the random return *R*. They do not know in advance what the fund/pool will earn. At the end of the year the annuitants will learn (or realize) their investment returns, and then split the gains among the surviving pool. Moreover, in the event that the investment earns a negative return—and loses money—the participants will share in the losses as well, but the effect will be mitigated by the mortality credits. Algebraically, the expected return will be the same (1 + R)/p - 1 > R. In fact, this concept is the foundation of an immediate *variable* annuity.

In practice, most insurance companies go one step further than the above (participating annuity) example and actually *guarantee* that the annuitant will receive the mortality credit enhancements, even if the mortality experience of the participants is better than expected. In other words, in the above-mentioned example for one-year fixed annuities, with an expected 20% mortality rate, the insurance company would guarantee that all survivors receive 31.25% on their money, regardless of whether or not 20% of the group died during the year.

Next, we will illustrate how risk pooling and the resulting longevity insurance works in practice, in the case of a life annuity rather than a tontine. Table #1b provides a set of hypothetical examples (since these numbers can change on a weekly basis). It states that a 65 year-old single female with \$100,000 can purchase an annuity that will provide her with \$630 per month for the rest of her life, no matter how long she lives. The same \$100,000 will buy an annuity that will provide a 65 year-old male with a greater income of \$730 per month for life. The additional \$100 per month, or 16%, that a male will receive is a direct result of his

lower life expectancy. That is, a group of 65 year-old men can expect to live 14 more years, while a group of 65 year-old females, on the other hand, can expect to live 20 more years, on average. Thus, if the money must last longer, the payments must be smaller.

Table 1b									
Sample Annuity Quotes									
\$100,000 buys monthly payments for life									
Current Age	Female	Male							
55	\$533	\$591							
65	\$630	\$730							
70	\$712	\$844							
75	\$832	\$1,008							
80	\$1,014	\$1,250							
Source: CANNEX Fina	ancial Exchanges, 2003								

With that in mind, if the 65-year-old exceeds the median life span of approximately 14 (20) more years, he (she) will end up earning a return that is greater than the average interest rate that was applied at the time of purchase. If he or she falls short of the median life span, the return will be inferior.

Another important aspect of life annuities is that the monthly payments that retirees can receive increase the longer they wait before buying the annuity. As noted previously, a 65-year-old male (female) can get \$730 (\$630) per month from a \$100,000 annuity. But if they waited another 10 years to make the purchase, until age 75, the male (female) would get \$1,008 (\$832), for life. That's an increase of approximately 35%, simply for deferring the purchase for 10 years.

Once again, the median life span is the key. At age 75, a male's median life span is approximately 83.5 years (86.5 for women). This translates into an average of 9 (12) more years of payments, as opposed to 14 (20) more years when they annuitize at age 65. The fewer years the age cohort is likely to live, the larger the monthly payments will be. So the lesson is: the longer you wait to annuitize, the more you will get per month.

The natural alternative to buying a life annuity is the do-it-yourself annuity using a systematic withdrawal plan (SWiP). A retiree can create and manage his or her own annuity stream. For example, the retiring 65-year-old male (female) can keep the \$100,000 invested in an IRA for the next few years and then start withdrawing a fixed monthly income of exactly

\$730 (\$630). That, you will remember, is the hypothetical annuity amount that the insurance company would have provided at age 65. But what if they live too long? Will their money last? Indeed, this do-it-yourself strategy runs a serious financial risk: under-funding retirement in the event of long-run inferior investment returns in conjunction with unexpected human longevity. This is exactly how and why a life annuity provides longevity insurance.

Here is an additional explanation of how age impacts the return from a fixed life annuity. Table #1c shows the internal rate of return (IRR) — a measure of profitability — from purchasing a life annuity with \$100,000 at various ages, assuming that you will live and receive payments until age 95.

	Table 1c									
You purchase a life annuity with \$100,000.										
What is your IRR, assuming you live to age 95?										
Purchase Age	Female	Male								
65	6.66%	8.24%								
75	8.22%	11.17%								
80	9.37%	13.54%								
85	10.00%	19.62%								

Table #1c states that if an 80 year-old male purchases a life annuity, and survives to age 95 — thus receiving \$1,250 per month for 15 years — he will earn an implicit annual return of 13.54% on his initial investment. It is implicit because when you discount 15 years of \$1,250 monthly payments, at a rate of 13.54% per year, you obtain the original \$100,000. Conceptually, this is equivalent to a \$100,000 home mortgage, amortized at 13.54% over a 15-year period, with monthly payments of \$1,250.

The same purchase at age 85 provides an even better yield — an implicit 19.62% annual return. A 19.62% return is very high and would clearly be difficult to beat using alternative investment classes. However, at age 65, the internal rate of return is much lower (8.24%), even with the assumption that you live to age 95.

In sum, we hope the above illustrations help the reader understand the benefits of longevity insurance and how these benefits can be translated into investment terms. A retiree who is receiving income from an immediate annuity earns a return on the order of magnitude displayed in Table #1c. This is why we advocate (eventual) annuitization in order to sustain and maintain a reasonable standard of living during retirement. Moreover and once again, only insurance companies can offer this form of risk pooling mechanism.

#### 3.) The Mechanics of a Guaranteed VPA: Protecting the Downside

There are many ways in which to construct or design a payout annuity instrument that offers downside protection to annuitants. The common denominator of all (possible) designs is that they offer an implicit <u>put option</u> on either: (i) actuarial mortality rates, (ii) interest rates or (iii) portfolio investment returns. Collectively we label them annuitization put options (APOs). For example, a \$10,000 premium deposited into an underlying fund by a 40 year-old can be attached to an explicit guarantee that if the contract is annuitized at age 65, the annuity will provide at least \$2,000 in annual income for the rest of the annuitant's life. In this case, the annuity payment (income) can fluctuate and be linked to the performance of an underlying fund. This guarantee would contain an explicit put option on investment returns and mortality rates.

Therefore, from our perspective, under the most general conditions, all guaranteed VPA (GVPA) structures can be analyzed within the following framework.

**Initial Retirement Income =** MAX[(D+C)/A,B] (Formula.1)

In this fundamental formula, the letter *D* denotes the original <u>deposit</u> premium, the letter *C* denotes the <u>cumulative</u> investment gains, the letter *A* denotes an <u>annuity</u> conversion factor and the letter *B* denotes a <u>base</u> income. Any and all of the three letters *A*,*B* or *C* can be guaranteed, projected or completely random. For example, a contributor might be guaranteed that their deposit premium will earn (or be credited with) at least 100% interest, so that C = D in the above formula, but only given an anticipation of what *A* and *B* will be. In this case, there really is no guarantee being provided since neither the right nor the left part of the MAX expression is given a lower bound in advance.

On the other hand, the contributor might be guaranteed that they will receive annual retirement income of at least 20% of their initial deposit, in which case B = 0.2D in the above formula, and a firm guarantee is being provided. On a slightly more esoteric level, the participant might be guaranteed that the annuity conversion factor will be A = 10, so that each

and every \$10 in the account (D+C) will generate at least \$1 of retirement income, etc. Note, also, that *C* itself can be zero or even negative.

The important point is that in order to analyze any guarantees that are embedded within a VPA, one must carefully represent the payoff based on (formula.1) and then read-off the values of A, B and C to determine whether they are simply anticipated or absolutely guaranteed. Note that these three variables themselves might contain their own guarantees -- for example A could be at least 20, or B could be at least \$1,000 – but all VPAs can still be expressed in the above manner.

Then, in subsequent years, the retirement income can increase (or decline) based on the performance of a reference investment basket or fund. We can express subsequent payments using a similar formula:

#### **Subsequent Retirement Income =** MAX[I+C,B], (Formula.2)

In this case, the new letter *I* denotes the previous year's retirement income and C denotes an aggregate investment return (net of fees and any adjustments) in the prior year, and B is the same guaranteed base. Note that C might be negative, which might reduce the subsequent year's income.

In the next section we conduct a number of simulation experiments to quantify the value of a guaranteed VPA relative to a non-guaranteed VPA assuming that there are no guarantees on the parameters C, that the parameter A is anticipated and that the parameter B is guaranteed. Note, once again, the distinction between a quantity that is random (investment returns), anticipated (mortality table to be used in converting the account into income) or guaranteed (minimum retirement income). For a more detailed understanding of the underlying formulas, we encourage the reader to consult the technical appendix.

### 4.) Numerical Examples and Analysis of One Possible Product Design

This section reports the results of extensive computer simulations that we conducted in an attempt to quantify the value of downside protection on an Immediate Variable Annuity (IVA). As mentioned earlier, all of our simulations were conducted with an assumed guaranteed structure that randomizes *C*, anticipates *A* and guarantees *B*, as per (formula.1). We hypothesize two plan-participants or investors, one of whom deposits \$10,000 in a regular VPA, which will be converted (a.k.a. annuitized) into an IVA, the other invests in a guaranteed VPA, which will be converted to a GIVA. Both individuals are charged the exact same level of fees and invest in the exact same underlying securities (or fund). The fees were identical for both types of accounts in order to isolate the value of the downside protection, rather than influence the outcome of the analysis by charging different fees. Further, both participants are also exposed to the same random growth-rate of the underlying fund, denoted by the letter *C*. They both convert (or exchange) their VPA into an IVA at the anticipated rate of *A* (for example, \$17 dollars per each dollar of lifetime income). Finally, the participant with the guarantee is promised the payment will never fall beyond the base *B*.

Note that the product design we are investigating in this section assumes that the guaranteed base B will be established at retirement. Specifically, in the basic cases, *B* will equal the maximum of a specified, age-dependent percentage of the initial deposit and 5% of the account value at retirement. Thus, the participant will "ratchet-up" the value of their initial guarantee, but only at the time of annuitization. This means that if the account value of the underlying fund increases substantially during the savings period and the original guaranteed base (which we denote by  $g_y$ ) is less than 5% applied to the retirement account value, it will be rational to step-up this guarantee. The technical appendix elaborates on this aspect of the guarantee.

Table #3 – which is in the last section of report -- lists the precise input variables that were used for each of the 25 case simulations we conducted. Given the scalable nature of the analysis, in all cases we assumed that \$10,000 was deposited into both accounts: the VPA (which upon annuitization becomes an IVA) and the GVPA (which upon annuitization becomes a GIVA). The subsequent tables list the input parameters (on top) and the output results (on the bottom) of each simulation. A number of input parameters are highlighted to emphasize the main focus of each simulation.

In Simulations #1-4, we assumed that the growth rate (denoted by *h*) is equal to 8%, and that the volatility (denoted by  $\sigma$ ) is equal to 15%. Asset-based fees were held constant at 50 basis points for the asset management fees (denoted by *f*<sub>A</sub>) and 80 basis points for the insurance expense fees (denoted by *f*<sub>G</sub>).

Based on our simulation results, we find that there is a significant probability that the initial income provided by the IVA will be less than that of the GIVA at the point of retirement, after deferral periods of 30, 20 and 10 years. This is what we mean by the probability of relative loss.

In Simulations #1-4, the individual who invests \$10,000 at age 45 and annuitizes the accumulated value at age 65 faces the highest relative risk of loss. In this case, according to the simulation results, the probability  $IVA_{65}$ <GIVA<sub>65</sub> is 31.09%. If the same \$10,000 is invested at age 35 and annuitization takes place at age 65, the probability of relative loss is 25.22%. The intuition for this result is as follows. Although the income base that is guaranteed at the time of the original investment is higher at age 35, the increased time span between investment and annuitization allows the initial deposit of \$10,000 to grow to a higher value. This, in turn, partially offsets the level of the higher guaranteed income factor.

Simulation #3 indicates that investing at age 55 and annuitizing at age 65 (after a 10 year deferral period) results in a probability of relative loss of 27.53%. The tables with Simulations #1-4 also provide loss probabilities for the later years of retirement. The relationship between the investment age and the probability of relative loss remains the same. The 35 year-old investor faces a chance of approximately 32.48% that the income provided by an IVA is less than that of the GIVA at age 70 (note that the opposite is not true, i.e., that 67.52% of the time the IVA > GIVA; in theory, the upside would be the same since they both use the same underlying fund); the 45 year-old faces a 34.24% chance and the 55 year-old investor – who annuitizes immediately – and there is an 18.75% chance that the IVA income will be less than the GIVA income at age 70. At age 75, the probability equals 15.95%, at age 80 – 12.90%, at age 85 – 10.25%, at age 90 – 9.02%, and finally at age 95 – 7.10%.

Simulations # 5a-5e were conducted using the growth rate and volatility parameters, based upon the historical returns of a generic diversified fund. In this particular case, the combination of a high return and a low volatility resulted in lower relative probabilities of loss. This is because the (better) performance of the fund is expected to trigger the income guarantee less frequently. In the subset containing Simulations #5a-5c we varied the fees for the contract of an investor assumed to annuitize immediately. In Simulations # 5d-5e, we assumed a 20 year deferral period – that is, an investment that is made at age 45 and annuitized at age 65 - and also varied the fees. As would be expected intuitively, higher combined fees resulted in marginally higher probabilities of relative loss.

We also conducted a set of simulations (#6-9) in which we examine the impact of increasing the risk of the underlying investment portfolio by changing the expected growth

and volatility parameters to 10% and 20%, respectively. In making this change, we discovered a reduced probability of relative loss during retirement years across all investment ages. This effect, not surprisingly, was most pronounced at younger investment ages. For example, as can be seen in Simulations #1 and 6, the risk of loss at retirement decreases by 10.27%. For a 45 year-old investor the risk at retirement drops by 9.56% and for a 55 year old investor – by 4.93%. However, if the investor annuitizes immediately upon investing at age 65, the probability that the income from the IVA is less than that of the GIVA at age 70 drops by only 0.2%.

Conversely, when we reduce the expected growth rate from 8% to 6% and reduce the volatility from 15% to 10%, we notice that the probability that the IVA<GIVA throughout the retirement years increases for all investment periods. This effect, again, is most noticeable when the initial deposit is made at age 35. Here, the loss of probability at retirement – at age 65 - rises from 25.22% (in Simulation #1) to 53.90% (in Simulation # 10). During the later retirement ages, for example age 70, the probability of loss rises from 32.48% to 53.79%. The alternative parameters do not have such a drastic impact when the investor immediately annuitizes his or her investment at age 65. For example, 5 years after retirement, at age 70, there is an 18.75% chance (Simulation #4) that the income from the IVA is less than the income from the GIVA under the initial parameters, whereas the probability increases by only 0.20% to 18.94% (Simulation #13) when the return and volatility are lowered.

The second metric that we focused on to assess the value of downside protection was the Option's Worth. The Option's Worth is a valuation procedure that attempts to estimate what it would cost an intermediary to hedge a particular guarantee using the market for derivative securities. This valuation procedure is at the heart of the famed Black-Scholes equation and is frequently used by investment banks and hedge funds to mark-to-market their trading position on a daily basis. This valuation procedure is also used for accounting purposes, for example, to estimate the cost of incentive stock options, which must be treated as expenses. Thus, the Option's Worth valuation algorithm is ubiquitous in financial markets and for this reason is quite appropriate in our context. For example, an Option's Worth of 140% means that the buyer is getting a value which is 40% more than what they are paying. Likewise, an Option's Worth of 95% implies that the buyer is losing 5% by acquiring this instrument. One can think of the Option's Worth as the generalized money's worth, except that the random nature of the underlying variables is taken into account explicitly. Mathematically, the Option's Worth is computed by projecting future cash flow benefits – net of any fees paid -- at a risk-adjusted rate and then discounting those cash flows at a risk-free rate. The precise mechanics of this calculation are discussed in greater detail in the technical appendix.

Note that in a properly functioning capital market, one would expect that the Option's Worth of any traded financial instrument is very close to 100%. Of course, when these instruments can not be sold short or are in the presence of transaction costs and other market restrictions, it is possible to see large discounts or premiums, which are represented by the Option's Worth. However, like any valuation methodology that is based on future assumptions, caution is warranted in interpreting the results. This is especially true within the context of long-term projections. At best, **an Option's Worth greater than 100% implies that the product adds relative economic value. And, an Option's Worth that is much smaller than 100% implies that economic value is being destroyed, both on a present value basis.** 

The set of simulations pertaining to the Option's Worth was comprised of Simulations #18-25. The (risk neutral) "expected benefits minus fees" quantity can be viewed as a proxy for the market value of the guarantee offered by the GIVA. A separate simulation was conducted in order to confirm that the value of the IVA is equal to the initial investment, when all fees are equal to zero. And indeed, the simulated expected value of the generic annuity is within \$15 of the original \$10,000 investment. The results of Simulations #19-22, where the insurance fee of 0.80% is charged, lead to the conclusion that the value of the downside protection is highest when the initial investment is made at age 35 and annuitized at age 65. This is compared to investing at ages 45, 55 and 65 while maintaining the same annuitization age. Specifically, the value of the GIVA at age 35 is approximately \$14,145, or 141% of the original investment for a 35 year old, \$12,664 or 127% of the initial investment for a 45 year old, \$10,403 or 104% for a 55 year old, but is only \$9,673 or 97% for a 65 year old.

Table #2 provides a summary of the relationship between the various inputs to the GIVA in relation to the (i) *probability of relative loss* and (ii) *Option's Worth* of the contract.

Table 2									
Wr	Is It Better or Worse? nat influences the Value of a GIN	/Α							
Input Factor	Probability of Relative Loss (GIVA better than IVA)	Options' Worth							
Higher Investment Volatility	Increases	Increase							
Better Investment Returns	Decrease	No Impact							
Purchase at Younger Age	Depends on Age & Horizon	Increases							
5% Ratchet at Retirement	Increases	Increases							
Projecting Mortality Table	Increases	Decreases							
Higher Asset-Based Fees	Increases	Decreases							
<b>Note:</b> In the GIVA vs. IVA comparison we are assuming that both products are identical in all aspects, except for the guaranteed base downside protection.									

### 5.) Conclusions and Final Remarks:

There appears to be universal agreement amongst financial economists and pension actuaries about the substantial social welfare benefits from payout (or immediate) annuity contracts. But the public and media have yet to embrace this risk-management instrument as being equally important as a well diversified retirement portfolio of stocks and bonds. Indeed, the global trend away from Defined Benefit (DB) and towards Defined Contribution (DC) plans, in conjunction with exceptionally low levels of voluntary annuitization, cry out for a new way—or revisiting old ways—of thinking about the provision of lifetime retirement income.

This paper promotes, advocates and explores the financial risk-and-return properties of a guaranteed variable payout annuity (GVPA). The GVPA would be acquired at a young age—and small premiums would be paid over a long period of time—but would begin paying at retirement. **The GVPA is a close relative of a DB pension and is intended for those who have an insufficient or no DB plan**, as an option within a DC (or 401k) style plan. The GVPA would entitle the holder to insure against the risk of outliving assets. We have provided a number of different metrics for quantifying the value and benefit of a GVPA compared to a generic VPA that does not offer any downside protection. We conclude by arguing that over long time horizons the implicit insurance benefit offered by such a structure can be substantial.

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#### **Technical Appendix:**

In the body of the report we conducted financial simulations to compute two distinct metrics for valuing the protection embedded within a Guaranteed Immediate Variable Annuity (GIVA), compared to a non-guaranteed or generic Immediate Variable Annuity (IVA). The first metric focused on the <u>Probability of Relative Loss</u> forecast that an IVA will generate less income compared to a GIVA, when both annuities are linked to the same underlying investment account. The second metric focused on the <u>Option's Worth</u>, which serves as a proxy for the amount capital markets would charge to re-insure the downside protection embedded within a GIVA, compared to an IVA which does not offer this protection. Computing the above-mentioned *probability of relative loss* and the *Option's Worth* requires distinct simulation inputs and outputs, although the underlying data generating models are similar. In this appendix we describe the precise methodology and assumptions behind these two metrics.

Our analysis starts at time *t*=0, when the participant/investor deposits a lump-sum  $W_0$  into a variable annuity account at age *y*. The value of the account (deposit) will grow at a stochastic annualized rate denoted by  $\eta$  (Greek letter eta), between the date of deposit and retirement age *x*>y at *t* = *T*. This annualized rate is gross of any fees.

For convenience, we assume that  $\eta$  is expressed using continuous compounding, which means that the random value of the variable annuity account (a.k.a. the original deposit) at time *T*, will be equal to and denoted by the symbol  $W_T := W_0 e^{\eta T}$ , before any asset management (AM) or mortality fees (ME) are deducted. Once we subtract these fees, the stochastic value of the variable annuity account at retirement will be equal to  $W_T := W_0 e^{(\eta - f_A - f_G)T}$ , where  $f_A$  denotes the annual AM fee and  $f_G$  denotes the annual ME fee, both expressed using continuous compounding. For all intents and purposes we will assume that the monthly compounding of fees is akin to continuous compounding since  $(1 + f / 12)^{12}$  is quite close to  $e^f$ .

In all the simulations, we assumed that the random growth rate  $\eta$  was Normally distributed with a (geometric) mean value of  $h = E[\eta]$  and a standard deviation (volatility) of  $\sigma/\sqrt{T-t}$ , where T-t denotes the calendar time that elapses between the deposit *t*, and retirement *T*. This obviously implies zero serial correlations between successive returns since

the returns are assumed independent. We are cognizant that our (simple) model might poorly describe the performance of a bond-heavy fund, but are reasonably confident that our results will not be meaningfully impacted when dealing with an equity-heavy fund. Stated differently, we believe that the uncertainty regarding the input parameters from long-term growth rates are more important – from a probabilistic perspective – compared to the difference between a 0% and 10% serial correlation.

In the *probability of relative loss* simulations we used values of h = 6%,8%,10% and volatility values of  $\sigma = 10\%,15\%,20\%$ . Note, also, that although:  $h = E[\eta]$ , the expected account value of  $W_T$  at retirement, is actually  $E[W_T] = W_0 e^{(h+0.5\sigma^2)T}$ . This is why the quantity  $h + 0.5\sigma^2$  is often labeled the arithmetic mean return and h is the geometric mean return. For example, if h = 8% and  $\sigma = 15\%$ , then  $0.08 + 0.5(0.15)^2 = 0.09125$  is the arithmetic mean return. We reported both numbers in all simulation results.

Now, let's move on to retirement, once the (guaranteed) variable payout annuity has been converted into a (guaranteed) immediate variable annuity. Let the symbol *IVA*<sub>i</sub> denote the annual income the annuitant receives from a generic IVA during the *i*th year of retirement. This quantity is stochastic and depends on the performance of the underlying fund in the (*i*-1)'th year of retirement. Likewise, let the symbol *GIVA*<sub>i</sub> denote the annual income the annuitant receives from the guaranteed IVA during the i'th year of retirement. This quantity is also stochastic, but will have a lower bound denoted by *G*, where *G* is determined and known at the point of retirement. The quantity *G* will automatically be set equal to the greater of  $g_y$  times the original deposit  $W_0$ , and  $g_{65} = 0.05$  times the account value at retirement  $W_T$ , where this new symbol  $g_y$  denotes the guaranteed income factor stipulated at the time of the deposit. For example, for the year 2005 baseline simulations, we used the following guaranteed income factor values:  $g_{35} = 0.2464$ ,  $g_{45} = 0.15536$ ,  $g_{55} = 0.08349$ .

The  $G = \max[g_{y}W_{0}, 0.05W_{r}]$  structure – which is the essence of the downside protection we are investigating in this report -- allows us to capture the option of the participant to stepup the guaranteed benefit just prior to annuitization. We also ran some cases with 0.04 and 0.06 instead of 0.05 to measure the sensitivity of the results to this so-called option. To understand the relationship between  $IVA_i$  and  $GIVA_i$ , we must first understand the recursive way in which the generic payment  $IVA_i$  is determined. Note that if  $UF_i$  denotes the gross return from the underlying fund (UF) during the i'th year of retirement, prior to any management fees, then:

$$IVA_{i+1} = (1 + M_i)IVA_i$$
, (eq.1)

where the new factor  $M_i$  is defined via:

$$M_{i} = \frac{UF_{i} + e^{-(f_{A} + f_{G})}}{1 + R} - 1, \qquad (eq.2)$$

The  $UF_i$  value is stochastic, and generated using the same distribution as in the accumulation phase;  $UF_i = e^n - 1$ . Note that  $M_i$  is obviously increasing with  $UF_i$ , but decreasing with the fees  $(f_A + f_G)$  and the assumed interest rate R. The value of  $M_i$  can be negative – and next year's income will decline -- if the gross returns from the underlying fund is not enough to overcome the fees and the hurdle rate R.

For example, if in the 5<sup>th</sup> year of retirement the income from the generic IVA was \$10,000 per annum and during the same year the underlying fund earned  $UF_5 = 11\%$  before asset management fees of  $f_A = 0.5\%$  and mortality expense fees of  $f_G = 0.8\%$ , then under an R = 3% assumed interest rate (AIR), the income from the generic IVA in the 6<sup>th</sup> year of retirement would be  $IVA_5 = 10000 \times (1 + 6.51\%)$ .

Now, moving on to compute the periodic payment under the guaranteed IVA, we must perform a two-step recursive procedure. First, at retirement, we compute the initial income from the GIVA using the following formula:

$$GIVA_1 = \max[G, W_T / a_x], \qquad (eq.3)$$

where *G* is the guaranteed base and  $a_x$  is the annuity factor, which is applied to the account value at retirement, time T. For example, under the Annuity 2000 (female) mortality table, the value of  $a_{65}$  at age 65 is 1000/55.78 = 17.927 per dollar of lifetime income, with a 20-year payment certain. Thus, if the account value at retirement was  $W_T = 50000$  in one of the simulation outcomes, then the initial GIVA income would be the greater of *G* and 50000/17.927 = 2789.1 dollars per year.

In the second and subsequent years of retirement, the annuity income from the GIVA will satisfy the following recursive equation:

$$GIVA_{i+1} = \max[(1 + M_i)IVA_i - L_i, G],$$
 (eq.4)

where  $M_i$  is the same income adjustment factor defined in equation (eq.2), but the new variable  $L_i$  is defined by:

$$L_i = \max[\sum_{j=1}^{i} (GIVA_j - IVA_j), 0]$$
 (eq.5)

Intuitively, one can think of  $L_i$  as a shadow account (or interest free loan), which keeps track of any excess payments that have been made to the GIVA annuitant, above and beyond payments made to the IVA annuitant. If this shadow account and the value of  $M_i$  are positive, then the GIVA annuitant will not receive the full increase in annuity payment relative to the previous year, until the shadow account balance is eliminated. Of course, there is always a floor in place, denoted by G, which creates a lower bound on payments.

To understand the mechanics of GIVA versus IVA, here is a simple three-period example. The assumed interest rate is R = 3%, the asset management fee is  $f_A = 0.5\%$  and the mortality expense fee is  $f_G = 0.8\%$ . Assume that G = 3000 dollars,  $W_T = 50000$  dollars and that  $a_{65} = 17.927$  at age 65, as per the Annuity 2000 (female) table. The non-guaranteed IVA payment will be equal to  $IVA_1 = 2789$  dollars, while the initial guaranteed payment will be  $GIVA_1 = 3000$  dollars. Thus, the shadow account starts off at a value of  $L_1 = 211$  dollars, since there has been an extra payment made to the protected annuitant. Assume further that  $UF_1 = -15\%$  in the first year of retirement. Thus,  $M_1 = -18.73\%$  according to equation (eq.2), and the IVA payment for the second year of retirement shrinks to  $IVA_2 = 2267$  dollars. The shadow account value has now increased to  $L_2 = 944$  dollars. Now, for the last part of this example, assume that  $UF_2 = +30\%$  in the second year of retirement. In this case,  $\{M_2 = 24.96\%\}$  and the IVA increases to  $IVA_3 = 2832$ . However, the GIVA payment will stay at \$3000, and the shadow account will increase to  $L_3 = 1112$  dollars.

With a numerical example behind us, and all the important symbols in place, we can put this all together to arrive at precise expressions for our two metrics of interest. The first is:

Probability of Relative Loss = 
$$\Pr[IVA_z \le GIVA_z|y]$$
 (eq.6)

This is the probability that IVA income will be lower than the GIVA income – and hence the protection was "worthwhile" – viewed from the perspective of someone aged y. The variable y can be age 35, 45, 55 or even 65 and the probability forecast will obviously depend on this conditioning age. The closer we are to the age z in question, the more information we have regarding whether  $\Pr[IVA_z \leq GIVA_z|y]$ . For example,  $\Pr[IVA_{75} \leq GIVA_{75}|45]$  denotes the probability – assuming the participant is currently 45 years old – that his/her IVA payment would be lower than the promised GIVA payment, at age 75. Likewise, the quantity  $\Pr[IVA_{75} \leq GIVA_{75}|55]$  is the same probability, but now we are conditioning on age 55. Of course, one is never certain whether they will actually be alive at age 75, which is why all our results report the probability of survival adjacent to the actual  $\Pr[IVA_z \leq GIVA_z|y]$  values. We will denote this quantity by  $({}_i P_y)$ . This is the probability that a *y*-year old will survive *i* years. For consistency, we used IAM 2000 values for this quantity as well.

Thus, by simulating values of  $UF_i$  and hence values of  $GIVA_i$  and  $IVA_i$ , we are able to count the number of times  $IVA_z \leq GIVA_z$  and divide this by the total number of simulations to arrive at an estimate for  $Pr[IVA_z \leq GIVA_z|y]$ . We used N = 10,000 simulation for all of our results.

Moving on, the second quantity of interest – the expected Option's Worth -- is computed in three stages. First, we quantify the discounted payoff via:

$$B = \sum_{i=1}^{120-t} (1+R)^{-(i+T-t)} (r_{-t+i} p_y) GIVA_i$$
 (eq.7)

The variable B is stochastic since we don't know the outcome of any of the  $GIVA_i$  payment values until after the returns  $UF_i$  have been observed. This is the present value of the benefits that one obtains from the guaranteed IVA, where the discounting takes place at the assumed interest rate R.

We then subtract-off any fees charged for this guaranteed IVA. This is calculated via the formula:

$$F = f_G \left( \sum_{j=1}^{T-t} (p_y) W_j (1+R)^{-j} + \sum_{i=1}^{120-x} (p_y) GIVA_i (a_{x+i}) (1+R)^{-(T-t+i)} \right), \quad (eq.8)$$

where the new quantity  $(a_{x+i})GIVA_i$  denotes the hypothetical value of the reserves supporting the annuity payment. This is necessary to keep track of the asset-base on which fees are being charged. One can think of  $(a_{x+i})GIVA_i$  as a "mark to market" value of the annuity. The insurance company must have at least this sum to back-up the annuity payments. The company earns fees of  $f_G$  on those payments. We could have also used  $(a_{x+i})IVA_i$  which would have increased the level of fees. Thus, our model provides a lower-bound.

The financial intuition for equation (eq.8) is as follows. The first item in the summation sign captures the value of the account between now and retirement. Each year the account value of  $W_j$  is charged a proportional mortality expense fee of  $f_G$ , for a total cash payment of  $f_G W_j$ . This amount must be discounted back to the present -- time *t*, age *y* -- which is the justification for the  $(1+R)^{-j}$  expression. This fee will only be charged to the account if the contributor is still alive, which is why we have placed  $({}_i P_y)$  in front. The second item in the summation sign captures the proportional mortality expense fees  $f_G$  that are charged in the payout phase. Note that this dollar-value is also adjusted for the probability of survival.

Finally, we divide the expected *benefits minus fees* by the original deposit  $W_0$  and subtract 1 to arrive at the (risk-neutral expected-value) *Option's Worth* of the GIVA.

Option's Worth = 
$$\frac{E^*[B] - E^*[F] - W_0}{W_0}$$
 (eq.9)

The only technical issue – which we are deliberately vague about – is that we are taking expectations of the random variables *B* and *F* under the assumption that the expect growth rate of  $h = E[\eta]$  is set equal to  $\ln(1+R) - 0.5\sigma^2$ , so that the expected account value after *j* periods, is exactly  $W_0(1+R)^j$ . This is known as a risk-neutral valuation. Note that the risk-neutral valuation ignores the asset management fees  $f_A$  since, although they are being paid by the participant, these fees should not impact the Option's Worth of the insurance component.

In the absence of any asset management fees  $f_A$  and mortality expense fees  $f_G$  fees, the Option's Worth of an IVA should be exactly 100% of the original deposit  $W_0$ , which we confirmed in a separate simulation. We then computed *Option's Worth* values at various deposit ages y = 35,45,55,65, assuming annuitization took place at age x = 65 under the same Annuity 2000 (female) mortality table.

Finally, we offer the following analytic expression for the initial *probability of relative loss* at the time of annuitization. This easy-to-use analytic expression provides an alternative to time consuming simulation techniques and can be used to calibrate as well as verify extensive simulation results. The assumption here is that the random (annualized) return, which we denoted by  $\eta$ , is Normally distributed, which means that  $e^{\eta}$  is Log-Normally distributed.

From the perspective of someone who is y-years of age and about to contribute  $W_0$  to the variable annuity account, the quantity  $\Pr[IVA_x < GIVA_x|y]$  is mathematically equivalent to  $\Pr[IVA_x < G|y]$ . But, since the definition of the minimally guaranteed income is via  $G = \max[0.05W_T, g_yW_0]$  and by definition  $W_T / a_x > 0.05W_T$ , we are left with the relationship:

$$\Pr[IVA_{x} < GIVA_{x} | y] = \Pr\left[\frac{W_{T}}{a_{x}} < g_{y}W_{0}\right].$$
 (eq.10)

Intuitively, the possible step-up at retirement might increase the magnitude of the variable G, but it will not impact the probability that the initial payment under the generic IVA will be lower than the guaranteed amount. Finally, since  $W_T$  is Log-Normally distributed, the probability can be obtained in closed-form as:

$$\Pr\left[\frac{W_T}{W_0} < (g_y)(a_x)\right] = \Pr\left[\mathbf{Z} < \frac{\ln[a_x] + \ln[g_y] - (h - f_A - f_G)(T - t)}{\sigma\sqrt{T - t}}\right] \quad (eq.11)$$

where Z is the standard (mean zero, variance one) Normal random variable. This function is easily available in Excel using the syntax *NormsDist(z)*. Thus, for example, if h = 8%,  $\sigma = 15\%$ ,  $g_{35} = 0.2464$ ,  $a_{65} = 17.927$  and the fees are  $f_A = 0.5\%$ ,  $f_G = 0.8\%$ , then according to equation (eq.11) the probability that a 35-year old contributor will experience an initial retirement income from GIVA that is greater than IVA is approximately 26.16%. This number is wellwithin the simulation margin of error which was reported in the body of the report.

## **Results Tables:**

	Table 3										
			Probability of I	Loss							
Sim #	I. Age/ R. Age	Growth rate/volatility <i>h</i> /σ	AM fees and ME fees f <sub>A</sub> / f <sub>G</sub>	Retirement Ratchet Assumption	Assumed Annuity Factor						
1	35/65	8%/15%	0.50%/0.80%	5%	\$17.93						
2	45/65	8%/15%	0.50%/0.80%	5%	\$17.93						
3	55/65	8%/15%	0.50%/0.80%	5%	\$17.93						
4	65/65	8%/15%	0.50%/0.80%	5%	\$17.93						
5a	65/65	9.57%/10.11%	0.50%/0.80%	5%	\$17.93						
5b	65/65	9.57%/10.11%	0.30%/0.80%	5%	\$17.93						
5c	65/65	9.57%/10.11%	0.75%/0.90%	5%	\$17.93						
5d	45/65	9.57%/10.11%	0.30%/0.80%	5%	\$17.93						
5e	45/65	9.57%/10.11%	0.75%/0.90%	5%	\$17.93						
6	35/65	10%/20%	0.50%/0.80%	5%	\$17.93						
7	45/65	10%/20%	0.50%/0.80%	5%	\$17.93						
8	55/65	10%/20%	0.50%/0.80%	5%	\$17.93						
9	65/65	10%/20%	0.50%/0.80%	5%	\$17.93						
10	35/65	6%/10%	0.50%/0.80%	5%	\$17.93						
11	45/65	6%/10%	0.50%/0.80%	5%	\$17.93						
12	55/65	6%/10%	0.50%/0.80%	5%	\$17.93						
13	65/65	6%/10%	0.50%/0.80%	5%	\$17.93						
14	35/65	8%/15%	0.50%/0.80%	4%	\$17.93						
15	55/65	8%/15%	0.50%/0.80%	4%	\$17.93						
16	35/65	8%/15%	0.50%/0.80%	6%	\$17.93						
17	55/65	8%/15%	0.50%/0.80%	6%	\$17.93						
			Option's Wo	rth							
18	65/65	1.90%/15%	0%/0%	5%	\$17.93						
19	35/65	1.90%/15%	0%/0.80%	5%	\$17.93						
20	45/65	1.90%/15%	0%/0.80%	5%	\$17.93						
21	55/65	1.90%/15%	0%/0.80%	5%	\$17.93						
22	65/65	1.90%/15%	0%/0.80%	5%	\$17.93						
23	35/65	1.90%/15%	0%/0.80%	5%	\$20.75						
24	45/65	1.90%/15%	0%/0.80%	5%	\$19.77						
25	55/65	1.90%/15%	0%/0.80%	5%	\$18.82						

#### Note: Interpretation of Results Tables

Here is how to interpret the results in the following tables. The upper portion of each table contains a listing of the input parameters in the simulation. For example, in the table for Simulation 1, we assumed the annuitization AIR was 3%, the asset based fee was 50 basis points for management fees and 80 basis points for insurance fees. The future investment growth rate of the underlying fund (UF) was assumed to be 8% per annum, with a volatility (standard deviation) of 15%, which leads to an arithmetic mean of 9.13%. The simulation assumed that at the point of annuitization the base guarantee would be ratcheted-up to 5% of the accumulated account value, if this value was greater than the original guarantee of 24.64% of the \$10,000 deposit.

Using these parameters, we found that the \$10,000 deposit grows to a median value of \$77,928 at retirement. When this median value is divided by the assumed annuity factor of \$17.93 per dollar of lifetime income, we arrive at a median initial income of \$4,347 per year. This can be contrasted with the median guaranteed base of \$3,896. In other words, most of the time the account value at retirement was sufficiently high – after 30 years of investing in the UF – so that the guaranteed base was less than the entitlement under the annuity factor. The table also illustrates the probabilities of relative loss (that the GIVA produces more income than the IVA) and the percentiles of the distribution of payments at various ages. Thus, for example, the figures in rows with the labels "GIVA at 65" and "IVA at 65" can be interpreted as follows. First, the left hand column lists the conditional survival probability – the probability that the 35 year old investor survives 30 more years to age 65. Next, from the perspective of a 35-year old, there is a 10% chance that an IVA will generate less than \$1,471 at retirement (for example if markets "stay crashed" during the next 30 years), but the GIVA generates at least \$2,464 of lifetime income, which is 24.64% of the original deposit of \$10,000. Finally, the right-hand column states the probability that the IVA will produce less income than the GIVA at retirement, which is equal to 25.22%. The remaining rows can be interpreted in the same manner, from the perspective of a 35 year-old. Finally, when relevant, the bottom row of the table displays an analytic confirmation of the simulation results obtained for the year of retirement. Please review the technical appendix for more information on this metric.

Simulation 1: Scenario Parameters										
System A.I.R.:		3%		Initia	al Investment	Age ( <i>y</i> ):		35		
System AM Fee:		0.50%	6	Ann	uitization Age	:		65		
System M&E Fe	e:	0.80%	6	Initia	al Investment:	1		\$1	0,000	
Expected Growth Rate:			6	Med	Account Val	ue at Ret:		\$7	7,928	
Arithmetic Mean	Rate:	9.13%	6	Assu	umed Annuity	Factor:		\$	17.93	
Standard Deviat	ion:	15.00	%	Med	ian Initial Ann	ual Income:		\$	4,347	
<b>Retirement Ratc</b>	het Assumption	n: 5%		Med	ian Guarantee	ed Base:		\$	3,896	
Scenario Results										
		Perce	Percent of Simulations Where Income Was Under:							
Survival Probability	Annuity Type	10%	25%		50%	75%	9	0%	Pr [Loss]	
	GIVA at 65	\$2,464	\$2,4	64	\$4,347	\$7,395	\$12	2,247	0.00%	
<sub>30</sub> <b>P</b> <sub>35</sub> = 94.19%	IVA at 65	\$1,471	\$2,4	52	\$4,347	\$7,395	\$12	2,247	25.22%*	
	GIVA at 70	\$2,464	\$2,7	96	\$5,332	\$9,528	\$16	6,088	0.00%	
<sub>35</sub> <b>P</b> <sub>35</sub> = 90.66%	IVA at 70	\$1,627	\$2,8	07	\$5,209	\$9,396	\$1	5,968	32.48%	
	GIVA at 75	\$2,464	\$3,2	64	\$6,384	\$12,082	\$2 <sup>-</sup>	1,493	0.00%	
40 <b>P</b> 35 = 85.09%	IVA at 75	\$1,800	\$3,2	46	\$6,244	\$11,936	\$2 <sup>-</sup>	1,493	25.85%	
	GIVA at 80	\$2,464	\$3,8	09	\$7,666	\$15,095	\$27	7,702	0.00%	
<sub>45</sub> <b>P</b> <sub>35</sub> = 75.88%	IVA at 80	\$2,031	\$3,7	84	\$7,581	\$15,027	\$27	7,772	20.63%	
	GIVA at 85	\$2,464	\$4,5	00	\$9,109	\$18,956	\$3	5,964	0.00%	
<sub>50</sub> <b>P</b> <sub>35</sub> = 61.50%	IVA at 85	\$2,304	\$4,4	66	\$9,038	\$18,894	\$3	5,951	16.90%	
	GIVA at 90	\$2,464	\$5,1	59	\$11,011	\$23,609	\$47	7,515	0.00%	
<sub>55</sub> <b>P</b> <sub>35</sub> = 41.87%	IVA at 90	\$2,528	\$5,1	53	\$10,964	\$23,607	\$47	7,515	14.21%	
	GIVA at 95	\$2,464	\$6,0	32	\$13,345	\$28,927	\$59	9,995	0.00%	
<sub>60</sub> <b>P</b> <sub>35</sub> = 21.64%	IVA at 95	\$2,881	\$6,0	14	\$13,374	\$28,931	\$59	9,995	11.81%	
*Analytic confirmation	ation: Pr [IVA <sub>65</sub> <0	GIVA <sub>65</sub>   y]=26.16	5%							

Simulation 2: Scenario Parameters											
System A.I.R.:		3%		Initia	al Investment	Age ( <i>y</i> ):			45		
System AM Fee:		0.50%	%	Ann	uitization Age	):		65			
System M&E Fe	e:	0.80%	%	Initia	al Investment:		\$1	0,000			
Expected Growt	h Rate:	8.00%			. Account Val	ue at Ret:		\$3	38,795		
Arithmetic Mean	Rate:	9.13%	%	Assı	umed Annuity	Factor:		\$	17.93		
Standard Deviat	ion:	15.00	%	Medi	ian Initial Ann	ual Income:		\$	2,164		
Retirement Ratc	het Assumption	n: 5%		Medi	ian Guarantee	ed Base:		\$	1,940		
Scenario Results											
		Perce	Percent of Simulations Where Income Was Under:								
Survival Probability	Annuity Type	10%	25%		50%	75%	9	0%	Pr [Loss]		
	GIVA at 65	\$1,554	\$1,55	54	\$2,164	\$3,441	\$5	,170	0.00%		
<sub>20</sub> <b>P</b> <sub>45</sub> = 94.77%	IVA at 65	\$897	\$1,37	'5	\$2,164	\$3,441	\$5	,170	31.09%*		
	GIVA at 70	\$1,554	\$1,55	64	\$2,714	\$4,431	\$6	,939	0.00%		
<sub>25</sub> <b>P</b> <sub>45</sub> = 91.22%	IVA at 70	\$1,005	\$1,57	'1	\$2,659	\$4,393	\$6	,905	34.24%		
	GIVA at 75	\$1,554	\$1,68	6	\$3,253	\$5,596	\$9	,216	0.00%		
<sub>30</sub> <b>P</b> <sub>45</sub> = 85.62%	IVA at 75	\$1,098	\$1,79	2	\$3,195	\$5,565	\$9	,200	27.27%		
	GIVA at 80	\$1,554	\$2,00	2	\$3,859	\$7,083	\$12	2,050	0.00%		
<sub>35</sub> <b>P</b> <sub>45</sub> = 76.35%	IVA at 80	\$1,193	\$2,07	'1	\$3,841	\$7,070	\$12	2,047	21.08%		
	GIVA at 85	\$1,554	\$2,32	28	\$4,617	\$8,757	\$1	5,644	0.00%		
40 <b>P</b> 45 = 61.88%	IVA at 85	\$1,339	\$2,38	57	\$4,612	\$8,740	\$1	5,640	17.05%		
	GIVA at 90	\$1,554	\$2,75	57	\$5,599	\$11,001	\$20	0,327	0.00%		
<sub>45</sub> <b>P</b> <sub>45</sub> = 42.13%	IVA at 90	\$1,482	\$2,80	)5	\$5,606	\$11,022	\$20	0,327	14.00%		
	GIVA at 95	\$1,554	\$3,19	3	\$6,757	\$13,707	\$20	6,211	0.00%		
<sub>50</sub> <b>P</b> <sub>45</sub> = 21.77%	IVA at 95	\$1,653	\$3,24	.3	\$6,757	\$13,723	\$26	6,211	11.91%		
*Analytic confirmation	ation: Pr [IVA <sub>65</sub> <	GIVA <sub>65</sub>   y]=31.90	)%				_				

Simulation 3: Scenario Parameters										
System A.I.R.:		3%		Initia	al Investment	Age ( <i>y</i> ):		55		
System AM Fee:		0.50%	%	Ann	uitization Age	:		65		
System M&E Fe	e:	0.80%	6	Initia	al Investment:			\$	0,000	
Expected Growt	Expected Growth Rate: 8.00%			Med	. Account Val	ue at Ret:		\$^	9,927	
Arithmetic Mean	Rate:	9.13%	6	Assı	umed Annuity	Factor:		\$	17.93	
Standard Deviat	ion:	15.00	%	Medi	ian Initial Ann	ual Income:		\$	1,111	
<b>Retirement Ratc</b>	het Assumption	n: 5%		Medi	ian Guarantee	ed Base:		:	\$996	
Scenario Results										
		Perce	Percent of Simulations Where Income Was Under							
Survival Probability	Annuity Type	10%	25%		50%	75%	9	0%	Pr [Loss]	
	GIVA at 65	\$835	\$83	5	\$1,111	\$1,535	\$2	,057	0.00%	
<sub>10</sub> <b>P</b> <sub>55</sub> = 96.22%	IVA at 65	\$600	\$803	3	\$1,111	\$1,535	\$2	,057	27.53%*	
	GIVA at 70	\$835	\$886	6	\$1,360	\$2,023	\$2	,854	0.00%	
<sub>15</sub> <b>P</b> <sub>55</sub> = 92.62%	IVA at 70	\$630	\$894	4	\$1,337	\$2,009	\$2	,847	30.53%	
	GIVA at 75	\$835	\$1,03	30	\$1,628	\$2,558	\$3	,861	0.00%	
<sub>20</sub> <b>P</b> <sub>55</sub> = 86.92%	IVA at 75	\$671	\$1,01	19	\$1,612	\$2,558	\$3	,863	23.92%	
	GIVA at 80	\$835	\$1,16	68	\$1,966	\$3,244	\$5	,073	0.00%	
<sub>25</sub> <b>P5</b> <sub>55</sub> =77.52%	IVA at 80	\$738	\$1,17	76	\$1,960	\$3,258	\$5	,075	18.41%	
	GIVA at 85	\$835	\$1,34	48	\$2,365	\$4,096	\$6	,695	0.00%	
<sub>30</sub> <b>P</b> <sub>55</sub> = 62.83%	IVA at 85	\$805	\$1,35	51	\$2,367	\$4,106	\$6	,695	14.70%	
	GIVA at 90	\$835	\$1,54	19	\$2,858	\$5,207	\$8	,935	0.00%	
<sub>35</sub> <b>P</b> <sub>55</sub> = 42.77%	IVA at 90	\$884	\$1,56	64	\$2,863	\$5,210	\$8	,935	12.04%	
	GIVA at 95	\$892	\$1,76	60	\$3,426	\$6,576	\$1 <sup>·</sup>	1,562	0.00%	
40 <b>P</b> 55 = 22.10%	IVA at 95	\$997	\$1,78	31	\$3,438	\$6,576	\$1 <sup>.</sup>	1,562	9.79%	
*Analytic confirmation	ation: Pr [IVA <sub>65</sub> <0	GIVA <sub>65</sub>   y]=28.70	)%							

Simulation 4: Scenario Parameters											
System A.I.R.:		3%	l	nitia	I Investment	Age ( <i>y</i> ):			65		
System AM Fee:		0.50%	% A	Annı	uitization Age	:		65			
System M&E Fee	e:	0.80%	6 I	nitia	nitial Investment:				0,000		
Expected Growt	h Rate:	8.00%	<mark>⁄~</mark> N	Med.	Account Val	ue at Ret:		\$	0,000		
Arithmetic Mean	Rate:	9.13%	/o 4	Annı	uity Factor:			\$	17.93		
Standard Deviat	ion:	15.00	<mark>% </mark>	Nedi	an Initial Ann	ual Income:		:	\$558		
<b>Retirement Ratc</b>	het Assumption	n: 5%	Ν	Nedi	an Guarantee	ed Base:			\$500		
Scenario Results											
		Perce	Percent of Simulations Where Income Was Under:								
Survival Probability	Annuity Type	10%	25%		50%	75%	9	0%	Pr [Loss]		
	GIVA at 65	\$558	\$558		\$558	\$558	\$	558	0.00%		
₀ <b>P</b> <sub>65</sub> = 100.00%	IVA at 65	\$558	\$558		\$558	\$558	\$	558	0.00%		
	GIVA at 70	\$500	\$513		\$668	\$845	\$1	,029	0.00%		
<sub>5</sub> <b>P</b> <sub>65</sub> = 96.25%	IVA at 70	\$437	\$535		\$669	\$845	\$1	,029	18.75%		
	GIVA at 75	\$500	\$551		\$810	\$1,117	\$1	,490	0.00%		
10 <b>P</b> 65 = 90.34%	IVA at 75	\$438	\$582		\$811	\$1,117	\$1	,490	15.95%		
	GIVA at 80	\$500	\$628		\$977	\$1,466	\$2	,094	0.00%		
<sub>15</sub> <b>P</b> <sub>65</sub> = 80.56%	IVA at 80	\$457	\$656		\$977	\$1,466	\$2	,094	12.90%		
	GIVA at 85	\$500	\$733		\$1,183	\$1,860	\$2	,850	0.00%		
<sub>20</sub> <b>P</b> <sub>65</sub> = 65.29%	IVA at 85	\$494	\$746		\$1,184	\$1,860	\$2	,850	10.25%		
	GIVA at 90	\$500	\$839		\$1,422	\$2,389	\$3	,854	0.00%		
<sub>25</sub> <b>P</b> <sub>65</sub> = 44.45%	IVA at 90	\$527	\$848		\$1,423	\$2,389	\$3	,854	9.02%		
	GIVA at 95	\$500	\$966		\$1,735	\$3,054	\$5	,145	0.00%		
<sub>30</sub> <b>P</b> <sub>65</sub> = 22.97%	IVA at 95	\$579	\$973		\$1,735	\$3,054	\$5	,145	7.10%		

Simulation 5a: Scenario Parameters											
System A.I.R.:		3%		Initia	l Investment	Age ( <i>y</i> ):			65		
System AM Fee:		0.50%	%	Ann	uitization Age	:		65			
System M&E Fee	e:	0.80%	/o	Initia	I Investment:	\$	10,000				
Expected Growt	h Rate:	9.57%	<mark>/o</mark>	Med	Account Val	ue at Ret:		\$10,000			
Arithmetic Mean	Rate:	10.08	%	Ann	uity Factor:			\$	17.93		
Standard Deviat	ion:	10.11	%	Med	ian Initial Ann	ual Income:			\$558		
Retirement Ratc	het Assumption	n: 5%	5% Median Guarar						\$500		
Scenario Results											
		Perce	Percent of Simulations Where Income Was Under:								
Survival Probability	Annuity Type	10%	25%	/ 0	50%	75%	9	0%	Pr [Loss]		
	GIVA at 65	\$558	\$55	8	\$558	\$558	\$	558	0.00%		
<sub>0</sub> <b>P</b> <sub>65</sub> = 100.00%	IVA at 65	\$558	\$55	8	\$558	\$558	\$	558	0.00%		
	GIVA at 70	\$533	\$62	2	\$727	\$850	\$	972	0.00%		
<sub>5</sub> <b>P</b> <sub>65</sub> = 96.25%	IVA at 70	\$541	\$62	3	\$727	\$850	\$	972	5.04%		
	GIVA at 75	\$626	\$76 <sup>-</sup>	7	\$949	\$1,185	\$1	,433	0.00%		
10 <b>P</b> 65 = 90.34%	IVA at 75	\$629	\$76 <sup>-</sup>	7	\$949	\$1,185	\$1	,433	2.48%		
	GIVA at 80	\$749	\$95	5	\$1,240	\$1,622	\$2	,065	0.00%		
<sub>15</sub> <b>P</b> <sub>65</sub> = 80.56%	IVA at 80	\$750	\$95	5	\$1,240	\$1,622	\$2	,065	1.10%		
	GIVA at 85	\$896	\$1,19	94	\$1,619	\$2,207	\$2	,930	0.00%		
<sub>20</sub> <b>P</b> <sub>65</sub> = 65.29%	IVA at 85	\$896	\$1,19	94	\$1,619	\$2,207	\$2	,930	0.59%		
	GIVA at 90	\$1,097	\$1,50	)5	\$2,108	\$2,996	\$4	,067	0.00%		
<sub>25</sub> <b>P</b> <sub>65</sub> = 44.45%	IVA at 90	\$1,097	\$1,50	)5	\$2,108	\$2,996	\$4	,067	0.21%		
	GIVA at 95	\$1,343	\$1,89	99	\$2,764	\$4,034	\$5	,739	0.00%		
<sub>30</sub> <b>P</b> <sub>65</sub> = 22.97%	IVA at 95	\$1,343	\$1,89	99	\$2,764	\$4,034	\$5	,739	0.09%		

Simulation 5b: Scenario Parameters											
System A.I.R.:		3%		Initia	I Investment	Age ( <i>y</i> ):			65		
System AM Fee:		0.30%	<mark>/o</mark>	Annu	uitization Age	:			65		
System M&E Fee	e:	0.80%	<mark>⁄₀</mark>	Initia	I Investment:	\$10,000					
Expected Growt	h Rate:	9.57%	<mark>// </mark>	Med.	Account Val		\$10,000				
Arithmetic Mean	Rate:	10.08	%	Annu	uity Factor:			\$	17.93		
Standard Deviat	ion:	10.11	%	Medi	an Initial Ann	ual Income:			\$558		
Retirement Ratc	het Assumption	n: 5%	5% Median Guaranteed Base:						\$500		
Scenario Results											
		Perce	Percent of Simulations Where Income Was Under:								
Survival Probability	Annuity Type	10%	25%		50%	75%	9	0%	Pr [Loss]		
	GIVA at 65	\$558	\$558		\$558	\$558	\$	558	0.00%		
<sub>0</sub> <b>P</b> <sub>65</sub> = 100.00%	IVA at 65	\$558	\$558		\$558	\$558	\$	558	0.00%		
	GIVA at 70	\$542	\$632		\$737	\$859	\$	986	0.00%		
<sub>5</sub> <b>P</b> <sub>65</sub> = 96.25%	IVA at 70	\$548	\$633		\$737	\$859	\$	986	4.44%		
	GIVA at 75	\$643	\$781		\$965	\$1,199	\$1	,458	0.00%		
10 <b>P</b> 65 = 90.34%	IVA at 75	\$645	\$781		\$965	\$1,199	\$1	,458	1.85%		
	GIVA at 80	\$776	\$976		\$1,273	\$1,659	\$2	,105	0.00%		
<sub>15</sub> <b>P</b> <sub>65</sub> = 80.56%	IVA at 80	\$778	\$976		\$1,273	\$1,659	\$2	,105	0.76%		
	GIVA at 85	\$940	\$1,24	0	\$1,678	\$2,287	\$2	,978	0.00%		
<sub>20</sub> <b>P</b> <sub>65</sub> = 65.29%	IVA at 85	\$940	\$1,24	0	\$1,678	\$2,287	\$2	,978	0.48%		
	GIVA at 90	\$1,152	\$1,57	2	\$2,222	\$3,131	\$4	,206	0.00%		
<sub>25</sub> <b>P</b> <sub>65</sub> = 44.45%	IVA at 90	\$1,152	\$1,57	2	\$2,222	\$3,131	\$4	,206	0.22%		
	GIVA at 95	\$1,442	\$2,00	0	\$2,891	\$4,234	\$5	,950	0.00%		
<sub>30</sub> <b>P</b> <sub>65</sub> = 22.97%	IVA at 95	\$1,442	\$2,00	0	\$2,891	\$4,234	\$5	,950	0.09%		

Simulation 5c: Scenario Parameters											
System A.I.R.:		3%		Initia	al Investment	Age ( <i>y</i> ):			65		
System AM Fee:		0.75%	/0	Ann	uitization Age	:		65			
System M&E Fee	e:	0.90%	/₀	Initia	al Investment:	\$10,000					
Expected Growt	h Rate:	9.57%	<mark>/o</mark>	Med	Account Val	ue at Ret:		\$10,000			
Arithmetic Mean	Rate:	10.08	%	Ann	uity Factor:			\$	17.93		
Standard Deviat	ion:	10.11	%	Medi	ian Initial Ann	ual Income:			\$558		
Retirement Ratc	het Assumption	n: 5%	5% Median Guaranteed Base:						\$500		
Scenario Results											
		Perce	Percent of Simulations Where Income Was Under:								
Survival Probability	Annuity Type	10%	25%	)	50%	75%	9	0%	Pr [Loss]		
	GIVA at 65	\$558	\$558	3	\$558	\$558	\$	558	0.00%		
<sub>0</sub> <b>P</b> <sub>65</sub> = 100.00%	IVA at 65	\$558	\$558	3	\$558	\$558	\$	558	0.00%		
	GIVA at 70	\$520	\$610	)	\$715	\$835	\$	960	0.00%		
<sub>5</sub> <b>P</b> <sub>65</sub> = 96.25%	IVA at 70	\$530	\$611	1	\$715	\$835	\$	960	6.09%		
	GIVA at 75	\$600	\$737	7	\$921	\$1,149	\$1	,391	0.00%		
10 <b>P</b> 65 = 90.34%	IVA at 75	\$607	\$737	7	\$921	\$1,149	\$1	,391	2.99%		
	GIVA at 80	\$703	\$903	3	\$1,186	\$1,555	\$1	,982	0.00%		
<sub>15</sub> <b>P</b> <sub>65</sub> = 80.56%	IVA at 80	\$706	\$903	3	\$1,186	\$1,555	\$1	,982	1.55%		
	GIVA at 85	\$833	\$1,11	4	\$1,514	\$2,068	\$2	,749	0.00%		
<sub>20</sub> <b>P</b> <sub>65</sub> = 65.29%	IVA at 85	\$835	\$1,11	4	\$1,514	\$2,068	\$2	,749	0.79%		
	GIVA at 90	\$1,012	\$1,38	80	\$1,940	\$2,732	\$3	,772	0.00%		
<sub>25</sub> <b>P</b> <sub>65</sub> = 44.45%	IVA at 90	\$1,012	\$1,38	80	\$1,940	\$2,732	\$3	,772	0.33%		
	GIVA at 95	\$1,220	\$1,70	)5	\$2,491	\$3,623	\$5	,136	0.00%		
<sub>30</sub> <b>P</b> <sub>65</sub> = 22.97%	IVA at 95	\$1,220	\$1,70	)5	\$2,491	\$3,623	\$5	,136	0.17%		

Simulation 5d: Scenario Parameters										
System A.I.R.:		3%		Initia	al Investment	Age ( <i>y</i> ):			45	
System AM Fee:	1	0.30%	6	Ann	uitization Age				65	
System M&E Fee	e:	0.80%	<mark>/o</mark>	Initia	al Investment:			\$10,000		
Expected Growt	h Rate:	9.57%	<mark>/o</mark>	Med	Account Val	ue at Ret:		\$54,830		
Arithmetic Mean	Rate:	10.08	%	Assı	umed Annuity	Factor:		\$17.93		
Standard Deviat	ion:	10.11 <sup>0</sup>	%	Med	ian Initial Ann	ual Income:		\$3,058		
Retirement Ratc	het Assumption	<b>ו:</b> 5%		Med	ian Guarantee	ed Base:		\$	2,741	
		S	cenario	o Res	sults					
Percent of Simulations Where Income Was Under:										
Survival Probability	Annuity Type	10% 25% 50% 75% 90% Pr [Los								
	GIVA at 65	\$1,700	\$2,2	54	\$3,058	\$4,175	\$5	,607	0.00%	
<sub>20</sub> <b>P</b> <sub>45</sub> = 94.77%	IVA at 65	\$1,700	\$2,2	54	\$3,058	\$4,175	\$5	,607	6.95%*	
	GIVA at 70	\$2,109	\$2,88	82	\$4,063	\$5,706	\$7	,910	0.00%	
<sub>25</sub> <b>P</b> <sub>45</sub> = 91.22%	IVA at 70	\$2,099	\$2,8	79	\$4,054	\$5,706	\$7	,920	7.31%	
	GIVA at 75	\$2,615	\$3,60	68	\$5,330	\$7,759	\$1 <sup>-</sup>	1,057	0.00%	
<sub>30</sub> <b>P</b> <sub>45</sub> = 85.62%	IVA at 75	\$2,615	\$3,60	66	\$5,330	\$7,759	\$1 <sup>-</sup>	1,057	3.01%	
	GIVA at 80	\$3,264	\$4,69	93	\$7,038	\$10,574	\$15	5,449	0.00%	
<sub>35</sub> <b>P</b> <sub>45</sub> = 76.35%	IVA at 80	\$3,265	\$4,69	95	\$7,038	\$10,574	\$15	5,449	1.31%	
	GIVA at 85	\$4,012	\$6,00	08	\$9,202	\$14,359	\$2 <sup>-</sup>	1,097	0.00%	
40 <b>P</b> 45 = 61.88%	IVA at 85	\$4,015	\$6,01	10	\$9,204	\$14,359	\$2 <sup>-</sup>	1,097	0.63%	
	GIVA at 90	\$5,059	\$7,70	00	\$12,161	\$19,364	\$29	9,716	0.00%	
<sub>45</sub> <b>P</b> <sub>45</sub> = 42.13%	IVA at 90	\$5,060	\$7,70	\$19,364	\$29	9,716	0.21%			
	GIVA at 95	\$6,394	\$9,9	18	\$16,066	\$26,243	\$40	0,805	0.00%	
50P45 = 21.77%   IVA at 95   \$6,395   \$9,918   \$16,067   \$26,243   \$40,805   0.15%										
*Analytic confirma	ation: Pr [IVA <sub>65</sub> <(	GIVA <sub>65</sub>   y]=6.94%	%							

Simulation 5e: Scenario Parameters										
System A.I.R.:		3%		Initia	al Investment	Age ( <i>y</i> ):			45	
System AM Fee:		0.75%	6	Ann	uitization Age	:			65	
System M&E Fee	e:	0.90%	6	Initia	al Investment:	1		\$10,000		
Expected Growt	h Rate:	9.57%	6	Med	Account Val	ue at Ret:		\$49,193		
Arithmetic Mean	Rate:	10.08	%	Assı	umed Annuity	Factor:		\$	17.93	
Standard Deviat	ion:	10.11	%	Med	ian Initial Ann	ual Income:		\$2,744		
Retirement Ratc	het Assumption	<b>1:</b> 5%		Med	ian Guarantee	ed Base:		\$	2,460	
		S	cenari	o Re	sults					
Percent of Simulations Where Income Was Under:										
Survival Probability	Annuity Type	10%	25%	6	50%	75%	9	0%	Pr [Loss]	
	GIVA at 65	\$1,554	\$2,0	17	\$2,744	\$3,770	\$4	,996	0.00%	
<sub>20</sub> <b>P</b> <sub>45</sub> = 94.77%	IVA at 65	\$1,553	\$2,0	17	\$2,744	\$3,770	\$4	,996	10.00%*	
	GIVA at 70	\$1,842	\$2,5	23	\$3,577	\$5,076	\$6	,864	0.00%	
<sub>25</sub> <b>P</b> <sub>45</sub> = 91.22%	IVA at 70	\$1,862	\$2,5	15	\$3,570	\$5,067	\$6	,862	9.36%	
	GIVA at 75	\$2,260	\$3,1	37	\$4,552	\$6,732	\$9	,437	0.00%	
<sub>30</sub> <b>P</b> <sub>45</sub> = 85.62%	IVA at 75	\$2,260	\$3,1	29	\$4,556	\$6,732	\$9	,442	4.84%	
	GIVA at 80	\$2,712	\$3,9	06	\$5,836	\$8,933	\$12	2,754	0.00%	
<sub>35</sub> <b>P</b> <sub>45</sub> = 76.35%	IVA at 80	\$2,714	\$3,9	10	\$5,838	\$8,933	\$12	2,754	2.47%	
	GIVA at 85	\$3,325	\$4,9	15	\$7,587	\$11,788	\$17	7,343	0.00%	
<sub>40</sub> <b>P</b> <sub>45</sub> = 61.88%	IVA at 85	\$3,332	\$4,9	28	\$7,586	\$11,788	\$17	7,343	1.18%	
	GIVA at 90	\$4,018	\$6,0	96	\$9,814	\$15,481	\$23	3,507	0.00%	
<sub>45</sub> <b>P</b> <sub>45</sub> = 42.13%	IVA at 90	\$4,022 \$6,098 \$9,814 \$15,481 \$23,507 0.								
	GIVA at 95	\$4,900	\$7,7	05	\$12,594	\$20,683	\$32	2,047	0.00%	
<sub>50</sub> <b>P</b> <sub>45</sub> = 21.77%	IVA at 95	\$4,908	\$7,7	05	\$12,594	\$20,683	\$32	2,047	0.30%	
*Analytic confirmation	ation: Pr [IVA <sub>65</sub> <0	GIVA <sub>65</sub> y]=10.80	)%							

Simulation 6: Scenario Parameters											
System A.I.R.:			3%	Initial Investmen	t Age ( <i>y</i> ):			35			
System AM Fee		0	.50%	Annuitization Ag	e:			65			
System M&E Fe	e:	0	.80%	Initial Investmen	t:		\$1	0,000			
Expected Growt	h Rate:	1(	0.00%	Med. Account Va	lue at Ret:		\$138,719				
Arithmetic Mear	n Rate:	12	2.00%	Assumed Annuit	y Factor:		\$17.93				
Standard Deviat	ion:	20	).00%	Median Initial An	nual Income:	\$7,738		,738			
Retirement Rato	het Assumptic	on:	5%	Median Guarante	ed Base:		\$6	,936			
			Scenario	Results							
Percent of Simulations Where Income Was Under:											
Survival Probability	Annuity Type	10%	25%	50%	75%	9	0%	Pr [Loss]			
	GIVA at 65	\$2,464	\$3,652	\$7,738	\$16,221	\$32	2,178	0.00%			
<sub>30</sub> <b>P</b> <sub>35</sub> = 94.19%	IVA at 65	\$1,873	\$3,652	\$7,738	\$16,221	\$32	2,178	14.95%*			
	GIVA at 70	\$2,464	\$4,771	\$10,546	\$23,616	\$48	3,643	0.00%			
<sub>35</sub> <b>P</b> <sub>35</sub> = 90.66%	IVA at 70	\$2,225	\$4,563	\$10,217	\$23,080	\$48	3,209	25.77%			
	GIVA at 75	\$2,464	\$6,137	\$14,023	\$33,494	\$72	2,283	0.00%			
40 <b>P</b> 35 = 85.09%	IVA at 75	\$2,667	\$5,888	\$13,799	\$33,216	\$7 <sup>-</sup>	1,546	19.23%			
	GIVA at 80	\$3,264	\$7,695	\$18,933	\$46,518	\$10	7,483	0.00%			
<sub>45</sub> <b>P</b> <sub>35</sub> = 75.88%	IVA at 80	\$3,334	\$7,360	\$18,695	\$46,118	\$10	7,332	14.32%			
	GIVA at 85	\$3,985	\$9,549	\$24,810	\$64,601	\$15	5,532	0.00%			
<sub>50</sub> <b>P</b> <sub>35</sub> = 61.50%	IVA at 85	\$4,037	\$9,447	\$24,608	\$64,672	\$15	5,609	11.15%			
	GIVA at 90	\$4,989	\$12,236	\$32,721	\$89,988	\$22	1,928	0.00%			
<sub>55</sub> <b>P</b> <sub>35</sub> = 41.87%	IVA at 90	\$4,886	\$4,886 \$12,240 \$32,721 \$90,006 \$221,928								
	GIVA at 95	\$6,043 \$15,612 \$44,273 \$125,364 \$328,021 0.00									
<sub>60</sub> P <sub>35</sub> = 21.64% IVA at 95 \$5,896 \$15,507 \$44,248 \$125,256 \$328,636 6.70%											
*Analytic confirm	ation: Pr [IVA <sub>65</sub> <	<giva<sub>65   y]=1</giva<sub>	5.23%								

Simulation 7: Scenario Parameters											
System A.I.R.:		3%		Initia	al Investment	Age ( <i>y</i> ):			45		
System AM Fee:		0.50%	6	Ann	uitization Age	:			65		
System M&E Fee	e:	0.80%	6	Initia	al Investment:			\$10,000			
Expected Growt	h Rate:	10.00	%	Med	. Account Val	ue at Ret:		\$57,186			
Arithmetic Mean	Rate:	12.00	%	Assı	umed Annuity	Factor:		\$	17.93		
Standard Deviat	ion:	20.00	%	Med	ian Initial Ann	ual Income:		\$3,190			
Retirement Ratc	het Assumption	n: 5%		Med	ian Guarantee	ed Base:		\$	2,859		
Scenario Results											
Percent of Simulations Where Income Was Under:											
Survival Probability	Annuity Type	10%	25%	6	50%	75%	9	0%	Pr [Loss]		
	GIVA at 65	\$1,554	\$1,7	35	\$3,190	\$5,814	\$10	0,161	0.00%		
<sub>20</sub> <b>P</b> <sub>45</sub> = 94.77%	IVA at 65	\$983	\$1,7	35	\$3,190	\$5,814	\$10,161		21.53%*		
	GIVA at 70	\$1,554	\$2,1	80	\$4,411	\$8,457	\$1	5,502	0.00%		
<sub>25</sub> <b>P</b> <sub>45</sub> = 91.22%	IVA at 70	\$1,143	\$2,0	99	\$4,223	\$8,340	\$1	5,355	28.98%		
	GIVA at 75	\$1,554	\$2,7	67	\$5,683	\$11,858	\$22	2,901	0.00%		
<sub>30</sub> <b>P</b> <sub>45</sub> = 85.62%	IVA at 75	\$1,328	\$2,6	85	\$5,584	\$11,770	\$22	2,901	21.22%		
	GIVA at 80	\$1,554	\$3,4	04	\$7,487	\$16,628	\$34	4,535	0.00%		
<sub>35</sub> <b>P</b> <sub>45</sub> = 76.35%	IVA at 80	\$1,663	\$3,34	43	\$7,479	\$16,620	\$34	4,508	15.60%		
	GIVA at 85	\$1,674	\$4,2	99	\$9,947	\$23,304	\$50	0,574	0.00%		
<sub>40</sub> <b>P</b> <sub>45</sub> = 61.88%	IVA at 85	\$1,940	\$4,2	28	\$9,947	\$23,306	\$50	0,574	11.84%		
	GIVA at 90	\$2,191	\$5,5	24	\$13,200	\$32,431	\$74	4,312	0.00%		
<sub>45</sub> <b>P</b> <sub>45</sub> = 42.13%	3%   IVA at 90   \$2,398   \$5,486   \$13,226   \$32,447   \$74,484   9.12%										
	GIVA at 95	\$2,767	\$6,9	33	\$17,633	\$45,258	\$10	9,188	0.00%		
<sub>50</sub> <b>P</b> <sub>45</sub> = 21.77%	<sub>50</sub> P <sub>45</sub> = 21.77% IVA at 95 \$2,882 \$6,960 \$17,664 \$45,262 \$109,188 7.25%										
*Analytic confirmation	ation: Pr [IVA <sub>65</sub> <0	GIVA <sub>65</sub> y]=21.18	3%								

Simulation 8: Scenario Parameters											
System A.I.R.:		3%		Initia	al Investment	Age ( <i>y</i> ):			55		
System AM Fee:		0.50%	6	Ann	uitization Age	:			65		
System M&E Fee	e:	0.80%	6	Initia	al Investment:	1		\$10,000			
Expected Growt	h Rate:	10.00	%	Med	. Account Val	ue at Ret:		\$24,321			
Arithmetic Mean	Rate:	12.00	%	Assı	umed Annuity	Factor:		\$17.93			
Standard Deviat	ion:	20.00	%	Med	ian Initial Ann	ual Income:		\$1,357			
Retirement Ratc	het Assumption	n: 5%		Med	ian Guarantee	ed Base:		\$	1,216		
		S	cenari	o Re	sults						
		Perce	nt of Sin	nulati	ons Where In	come Was Un	der:				
Survival Probability	Annuity Type	10%	25%	6	50%	75%	9	0%	Pr [Loss]		
	GIVA at 65	\$835	\$87	7	\$1,357	\$2,102	\$3	,128	0.00%		
<sub>10</sub> <b>P</b> <sub>55</sub> = 96.22%	IVA at 65	\$593	\$87	7	\$1,357	\$2,102	\$3	,128	22.60%*		
	GIVA at 70	\$835	\$1,0	99	\$1,871	\$3,102	\$4	,935	0.00%		
<sub>15</sub> <b>P</b> <sub>55</sub> = 92.62%	IVA at 70	\$656	\$1,0	70	\$1,821	\$3,081	\$4	,936	27.11%		
	GIVA at 75	\$835	\$1,3	37	\$2,444	\$4,502	\$7	,809	0.00%		
<sub>20</sub> <b>P</b> <sub>55</sub> = 86.92%	IVA at 75	\$755	\$1,3	06	\$2,422	\$4,497	\$7	,809	19.22%		
	GIVA at 80	\$835	\$1,6	32	\$3,192	\$6,416	\$1 <sup>-</sup>	1,941	0.00%		
<sub>25</sub> <b>P5</b> <sub>55</sub> =77.52%	IVA at 80	\$890	\$1,6	04	\$3,188	\$6,425	\$1 <sup>-</sup>	1,941	14.51%		
	GIVA at 85	\$975	\$2,0	26	\$4,226	\$9,073	\$18	3,020	0.00%		
<sub>30</sub> <b>P</b> <sub>55</sub> = 62.83%	IVA at 85	\$1,050	\$2,0	42	\$4,237	\$9,075	\$18	3,020	10.75%		
	GIVA at 90	\$1,166	\$2,4	90	\$5,680	\$12,836	\$2	7,066	0.00%		
<sub>35</sub> <b>P</b> <sub>55</sub> = 42.77%	IVA at 90	at 90 \$1,216 \$2,529 \$5,719 \$12,838 \$27,066 7.98									
	GIVA at 95	\$1,431	\$3,2	07	\$7,550	\$18,202	\$39	9,608	0.00%		
40 <b>P</b> 55 = 22.10%	40P55 = 22.10% IVA at 95 \$1,499 \$3,222 \$7,556 \$18,202 \$39,608 6.21%										
*Analytic confirmation	ation: Pr [IVA <sub>65</sub> <0	GIVA <sub>65</sub> y]=21.18	3%								

Simulation 9: Scenario Parameters												
System A.I.R.:		3%	li	nitia	I Investment	Age ( <i>y</i> ):			65			
System AM Fee:		0.50%	<b>A b</b>	۹nn	uitization Age	:			65			
System M&E Fee	e:	0.80%	6 li	nitia	I Investment:			\$10,000				
Expected Growt	h Rate:	10.00	<mark>% </mark>	/led.	Account Val	ue at Ret:		\$10,000				
Arithmetic Mean	Rate:	12.00	%	Annu	uity Factor:			\$17.93				
Standard Deviat	ion:	20.00	<mark>% </mark>	ual Income:	\$558							
Retirement Ratc	het Assumption	n: 5%	Ν	/ledi	an Guarantee	ed Base:			\$500			
Scenario Results												
Percent of Simulations Where Income Was Under:												
Survival Probability	Annuity Type	Percent of Simulations Where Income Was Under:   10% 25% 50% 75% 90% Pr [Loss]										
	GIVA at 65	\$558	\$558		\$558	\$558	\$	558	0.00%			
₀ <b>P</b> <sub>65</sub> = 100.00%	IVA at 65	\$558	\$558		\$558	\$558	\$	558	0.00%			
	GIVA at 70	\$500	\$521		\$744	\$1,009	\$1	,314	0.00%			
₅ <b>P</b> 65 = 96.25%	IVA at 70	\$420	\$552		\$747	\$1,010	\$1	,314	18.54%			
	GIVA at 75	\$500	\$621		\$1,009	\$1,542	\$2	,256	0.00%			
10 <b>P</b> 65 = 90.34%	IVA at 75	\$441	\$650		\$1,011	\$1,542	\$2	,256	14.02%			
	GIVA at 80	\$500	\$765		\$1,350	\$2,276	\$3	,576	0.00%			
<sub>15</sub> <b>P</b> <sub>65</sub> = 80.56%	IVA at 80	\$485	\$785		\$1,350	\$2,276	\$3	,576	10.69%			
	GIVA at 85	\$500	\$957		\$1,784	\$3,261	\$5	,583	0.00%			
<sub>20</sub> <b>P</b> <sub>65</sub> = 65.29%	IVA at 85	\$542	\$973		\$1,784	\$3,261	\$5	,583	8.47%			
	GIVA at 90	\$500	\$1,197	,	\$2,375	\$4,620	\$8	,562	0.00%			
<sub>25</sub> <b>P</b> <sub>65</sub> = 44.45%	IVA at 90	\$647 \$1,208 \$2,375 \$4,620 \$8,562 6.54%										
	GIVA at 95	\$673	\$1,510	)	\$3,156	\$6,689	\$12	2,775	0.00%			
<sub>30</sub> <b>P</b> <sub>65</sub> = 22.97%	IVA at 95	\$766	\$1,518	3	\$3,156	\$6,689	\$12	2,775	5.02%			

Simulation 10: Scenario Parameters												
System A.I.R.:			3	3%	Initi	ial Investment	Age ( <i>y</i> ):			35		
System AM Fee:			0.	50%	Anr	nuitization Age				65		
System M&E Fee	e:		0.8	80%	Initi	ial Investment:			\$10	0,000		
Expected Growt	h Rate:		6.0	00%	Med	d. Account Val	ue at Ret:		\$41,781			
Arithmetic Mean	Rate:		6.5	50%	Ass	umed Annuity	Factor:		\$17.93			
Standard Deviat	ion:		10.	.00%	Med	dian Initial Ann	ual Income:		\$2,330			
Retirement Ratc	het Assumptic	on:	5	5%	Med	dian Guarantee	ed Base:		\$2	,464		
				Scenari	o Re	sults						
Percent of Simulations Where Income Was Under:												
Survival Probability	Annuity Type	10	)%	25%		50%	75%	9	0%	Pr [Loss]		
	GIVA at 65	\$2,4	464	\$2,464		\$2,464	\$3,398	\$4	,672	0.00%		
<sub>30</sub> <b>P</b> <sub>35</sub> = 94.19%	IVA at 65	\$1, <sup>-</sup>	152	\$1,607	,	\$2,330	\$3,398	\$4	,672	53.90%*		
	GIVA at 70	\$2,4	464	\$2,464	Ļ	\$2,464	\$3,822	\$5	,402	0.00%		
<sub>35</sub> <b>P</b> <sub>35</sub> = 90.66%	IVA at 70	\$1, <sup>-</sup>	188	\$1,714	Ļ	\$2,560	\$3,790	\$5	,356	53.79%		
	GIVA at 75	\$2, <b>4</b>	464	\$2,464	Ļ	\$2,669	\$4,285	\$6	,155	0.00%		
40 <b>P</b> 35 = 85.09%	IVA at 75	\$1,2	231	\$1,793	3	\$2,794	\$4,240	\$6	,119	47.93%		
	GIVA at 80	\$2, <del>4</del>	464	\$2,464	Ļ	\$2,952	\$4,754	\$7	,071	0.00%		
<sub>45</sub> <b>P</b> <sub>35</sub> = 75.88%	IVA at 80	\$1,2	263	\$1,910	)	\$3,028	\$4,732	\$7	,058	42.96%		
	GIVA at 85	\$2,4	464	\$2,464	Ļ	\$3,213	\$5,301	\$8	,010	0.00%		
<sub>50</sub> <b>P</b> <sub>35</sub> = 61.50%	IVA at 85	\$1,3	306	\$2,028	3	\$3,317	\$5,294	\$8	,010	37.92%		
	GIVA at 90	\$2, <b>4</b>	464	\$2,464	Ļ	\$3,522	\$5,904	\$9	,296	0.00%		
<sub>55</sub> <b>P</b> <sub>35</sub> = 41.87%	IVA at 90	\$1,3	361	\$2,154		\$3,595	\$5,900	\$9	,278	34.43%		
	GIVA at 95	\$2,4	464	\$2,464	\$6,617	\$10,511 (		0.00%				
<sub>60</sub> <b>P</b> <sub>35</sub> = 21.64%	<sub>60</sub> P <sub>35</sub> = 21.64% IVA at 95 \$1,421 \$2,299 \$3,895 \$6,609 \$10,513 31.18%											
*Analytic confirmation	ation: Pr [IVA <sub>65</sub> <	GIVA <sub>6</sub>	₅ y]=55	5.49%								

Simulation 11: Scenario Parameters											
System A.I.R.:		3%		Initia	al Investment	Age ( <i>y</i> ):			45		
System AM Fee:		0.50%	6	Ann	uitization Age	:			65		
System M&E Fee	e:	0.80%	6	Initia	al Investment:	1		\$10,000			
Expected Growt	h Rate:	6.00%	6	Med	Account Val	ue at Ret:		\$25,846			
Arithmetic Mean	Rate:	6.50%	6	Assı	umed Annuity	Factor:		\$17.93			
Standard Deviat	ion:	10.00	%	Med	ian Initial Ann	ual Income:		\$1,442			
Retirement Ratc	het Assumption	n: 5%		Med	ian Guarantee	ed Base:		\$	1,554		
		S	cenario	o Res	sults						
		Percei	nt of Sin	nulati	ons Where In	come Was Un	der:				
Survival Probability	Annuity Type	10%	25%	6	50%	75%	9	0%	Pr [Loss]		
	GIVA at 65	\$1,554	\$1,5	54	\$1,554	\$1,947	\$2	2,556	0.00%		
<sub>20</sub> <b>P</b> <sub>45</sub> = 94.77%	IVA at 65	\$804	\$1,0	65	\$1,442	\$1,947	\$2	2,556	56.66%*		
	GIVA at 70	\$1,554	\$1,5	54	\$1,554	\$2,221	\$3	,021	0.00%		
<sub>25</sub> <b>P</b> <sub>45</sub> = 91.22%	IVA at 70	\$821	\$1,1	12	\$1,571	\$2,203	\$3	,008	54.06%		
	GIVA at 75	\$1,554	\$1,5	54	\$1,615	\$2,507	\$3	,469	0.00%		
<sub>30</sub> <b>P</b> <sub>45</sub> = 85.62%	IVA at 75	\$848	\$1,1	85	\$1,723	\$2,490	\$3	,465	47.32%		
	GIVA at 80	\$1,554	\$1,5	54	\$1,773	\$2,799	\$3	,964	0.00%		
<sub>35</sub> <b>P</b> <sub>45</sub> = 76.35%	IVA at 80	\$860	\$1,24	48	\$1,860	\$2,795	\$3	,969	41.63%		
	GIVA at 85	\$1,554	\$1,5	54	\$1,920	\$3,110	\$4	,591	0.00%		
<sub>40</sub> <b>P</b> <sub>45</sub> = 61.88%	IVA at 85	\$897	\$1,3°	12	\$2,016	\$3,104	\$4	,582	37.49%		
	GIVA at 90	\$1,554	\$1,5	54	\$2,113	\$3,441	\$5	5,218	0.00%		
<sub>45</sub> <b>P</b> <sub>45</sub> = 42.13%	IVA at 90	\$912	\$1,3	82	\$2,194	\$3,456	\$5	,219	33.81%		
	GIVA at 95	\$1,554	\$1,5	54	\$2,338	\$3,865	\$6	6,018	0.00%		
<sub>50</sub> <b>P</b> <sub>45</sub> = 21.77%	<sub>50</sub> P <sub>45</sub> = 21.77% IVA at 95 \$970 \$1,459 \$2,405 \$3,864 \$6,018 30.32%										
*Analytic confirmation	ation: Pr [IVA <sub>65</sub> <0	GIVA <sub>65</sub> y]=57.48	3%								

Simulation 12: Scenario Parameters											
System A.I.R.:		3%		Initia	l Investment	Age ( <i>y</i> ):			55		
System AM Fee:		0.50%	6	Annu	uitization Age	:			65		
System M&E Fee	e:	0.80%	6	Initia	I Investment:			\$10,000			
Expected Growt	h Rate:	6.00%	/o	Med.	Account Val	ue at Ret:		\$16,016			
Arithmetic Mean	Rate:	6.50%	6	Assı	umed Annuity	Factor:		\$17.93			
Standard Deviat	ion:	10.00	%	Medi	ian Initial Ann	ual Income:		:	\$893		
Retirement Ratc	het Assumption	n: 5%		Medi	ian Guarantee	ed Base:			\$835		
		S	cenario	Res	sults						
Percent of Simulations Where Income Was Under:											
Survival Probability	Annuity Type	10%   25%   50%   75%   90%   Pr     [Loss]   [Loss]<									
	GIVA at 65	\$835	\$835	5	\$893	\$1,118	\$1	,351	0.00%		
<sub>10</sub> <b>P</b> <sub>55</sub> = 96.22%	IVA at 65	\$594	\$722	2	\$893	\$1,118	\$1	,351	41.93%*		
	GIVA at 70	\$835	\$835	5	\$979	\$1,285	\$1	,630	0.00%		
<sub>15</sub> <b>P</b> <sub>55</sub> = 92.62%	IVA at 70	\$591	\$750	)	\$979	\$1,280	\$1	,627	40.73%		
	GIVA at 75	\$835	\$835	5	\$1,063	\$1,449	\$1	,913	0.00%		
<sub>20</sub> <b>P</b> <sub>55</sub> = 86.92%	IVA at 75	\$596	\$782	2	\$1,059	\$1,442	\$1	,913	35.52%		
	GIVA at 80	\$835	\$835	5	\$1,155	\$1,643	\$2	,213	0.00%		
<sub>25</sub> <b>P5</b> <sub>55</sub> =77.52%	IVA at 80	\$598	\$823	3	\$1,158	\$1,641	\$2	,215	30.75%		
	GIVA at 85	\$835	\$835	5	\$1,251	\$1,851	\$2	,603	0.00%		
<sub>30</sub> <b>P</b> <sub>55</sub> = 62.83%	IVA at 85	\$610	\$869	)	\$1,259	\$1,852	\$2	,603	26.46%		
	GIVA at 90	\$835	\$847	7	\$1,363	\$2,066	\$2	,985	0.00%		
<sub>35</sub> <b>P</b> <sub>55</sub> = 42.77%	IVA at 90	\$631	\$919	)	\$1,374	\$2,068	\$2	,985	23.13%		
	GIVA at 95	\$835	\$907	7	\$1,491	\$2,317	\$3	,434	0.00%		
40 <b>P</b> 55 = 22.10%	IVA at 95	\$655	\$962	2	\$1,503	\$2,319	\$3	,434	20.88%		
*Analytic confirmation	ation: Pr [IVA <sub>65</sub> <0	GIVA <sub>65</sub>   y]=41.65	5%								

Simulation 13: Scenario Parameters											
System A.I.R.:		3%	1	nitia	I Investment	Age ( <i>y</i> ):			65		
System AM Fee:		0.50%	% <b>/</b>	Annı	uitization Age	:			65		
System M&E Fe	e:	0.80%	% Ι	nitia	I Investment:			\$	10,000		
Expected Growt	h Rate:	6.00%	<mark>%</mark>	Med.	Account Val	ue at Ret:		\$10,000			
Arithmetic Mean	Rate:	6.50%	% <b>/</b>	Annı	uity Factor:			\$17.93			
Standard Deviat	ion:	10.00	<mark>% </mark>	Medi	an Initial Ann	ual Income:		\$558			
Retirement Ratc	het Assumption	n: 5%	I	Medi	an Guarantee	ed Base:			\$500		
		S	cenario	Res	sults						
Percent of Simulations Where Income Was Under:											
Survival Probability	Annuity Type	10%	25%		50%	75%	9	0%	Pr [Loss]		
	GIVA at 65	\$558	\$558		\$558	\$558	\$	558	0.00%		
₀ <b>P</b> <sub>65</sub> = 100.00%	IVA at 65	\$558	\$558		\$558	\$558	\$	558	0.00%		
	GIVA at 70	\$500	\$513		\$607	\$709	\$	810	0.00%		
<sub>5</sub> <b>P</b> <sub>65</sub> = 96.25%	IVA at 70	\$458	\$523		\$609	\$709	\$	810	18.94%		
	GIVA at 75	\$500	\$513		\$663	\$821	\$	997	0.00%		
10 <b>P</b> 65 = 90.34%	IVA at 75	\$440	\$536		\$664	\$821	\$	997	18.53%		
	GIVA at 80	\$500	\$528		\$721	\$940	\$1	,210	0.00%		
<sub>15</sub> <b>P</b> <sub>65</sub> = 80.56%	IVA at 80	\$440	\$553		\$721	\$940	\$1	,210	17.07%		
	GIVA at 85	\$500	\$552		\$785	\$1,067	\$1	,402	0.00%		
<sub>20</sub> <b>P</b> <sub>65</sub> = 65.29%	IVA at 85	\$441	\$581		\$786	\$1,067	\$1	,402	15.92%		
	GIVA at 90	\$500	\$586		\$856	\$1,206	\$1	,648	0.00%		
<sub>25</sub> <b>P</b> <sub>65</sub> = 44.45%	IVA at 90	\$450	\$609		\$857	\$1,206	\$1	,648	13.99%		
	GIVA at 95	\$500	\$623		\$935	\$1,366	\$1	,900	0.00%		
<sub>30</sub> P <sub>65</sub> = 22.97% <b>IVA at 95</b> \$465 \$643 \$935 \$1,366 \$1,900 12.39%											

Simulation 14: Scenario Parameters											
System A.I.R.:			3	3%	Initi	ial Investment	Age ( <i>y</i> ):			35	
System AM Fee	:		0.	50%	Anr	nuitization Age	:			65	
System M&E Fe	e:		0.8	80%	Initi	ial Investment:			\$10	0,000	
Expected Growt	th Rate:		8.	00%	Med	d. Account Val	ue at Ret:		\$76,031		
Arithmetic Mear	n Rate:		9.	13%	Ass	sumed Annuity	Factor:		\$17.93		
Standard Deviat	tion:		15	.00%	Med	dian Initial Ann	ual Income:		\$4,241		
Retirement Rate	chet Assumpt	ion:	2	4%	Med	dian Guarantee	ed Base:		\$3	,041	
Scenario Results											
			Perc	cent of Sim	nulati	ions Where Inc	ome Was Ur	nder:			
Survival Probability	Annuity Type	1	0%	25%		50%	75%	9	0%	Pr [Loss]	
	GIVA at 65	\$2	\$2,464 \$2,464			\$4,241	\$7,372	\$12,073		0.00%	
<sub>30</sub> <b>P</b> <sub>35</sub> = 94.19%	IVA at 65	\$1	,445	\$2,392		\$4,241	\$7,372	\$12	2,073	26.12%*	
	GIVA at 70	\$2	.,464	\$2,633	3	\$5,075	\$9,207	\$15	5,810	0.00%	
<sub>35</sub> <b>P</b> <sub>35</sub> = 90.66%	IVA at 70	\$1	,598	\$2,784	Ļ	\$5,047	\$9,179	\$15	5,802	24.83%	
	GIVA at 75	\$2	,464	\$3,153	3	\$6,127	\$11,729	\$20	0,762	0.00%	
40 <b>P</b> 35 = 85.09%	IVA at 75	\$1	,794	\$3,248	3	\$6,090	\$11,669	\$20	),712	20.51%	
	GIVA at 80	\$2	.,464	\$3,678	3	\$7,421	\$14,694	\$26	6,502	0.00%	
<sub>45</sub> <b>P</b> <sub>35</sub> = 75.88%	IVA at 80	\$1	,973	\$3,725	5	\$7,392	\$14,694	\$26	6,449	17.45%	
	GIVA at 85	\$2	2,464	\$4,252	2	\$8,980	\$18,383	\$34	1,331	0.00%	
<sub>50</sub> <b>P</b> <sub>35</sub> = 61.50%	IVA at 85	\$2	2,230	\$4,345	5	\$8,960	\$18,406	\$34	4,331	14.35%	
	GIVA at 90	\$2	.,464	\$5,042	2	\$10,768	\$23,275	\$44	4,311	0.00%	
<sub>55</sub> <b>P</b> <sub>35</sub> = 41.87%	IVA at 90	\$2	,497	\$5,076	6	\$10,790	\$23,273	\$44	1,311	11.92%	
	GIVA at 95	\$2	2,464	\$5,955	5	\$12,988	\$28,997	\$57	7,179	0.00%	
60 <b>P</b> <sub>35</sub> = 21.64% <b>IVA at 95</b> \$2,826 \$5,952 \$12,999 \$28,995 \$57,179 9.76%										9.76%	
*Analytic confirm	ation: Pr [IVA <sub>6</sub>	₅ <giva< th=""><th>A<sub>65</sub>   y]=26</th><th>6.16%</th><th></th><th></th><th></th><th></th><th></th><th></th></giva<>	A <sub>65</sub>   y]=26	6.16%							

Simulation 15: Scenario Parameters												
System A.I.R.:		3%		Initia	al Investment	Age ( <i>y</i> ):			55			
System AM Fee:		0.50%	6	Ann	uitization Age	:			65			
System M&E Fee	e:	0.80%	6	Initia	al Investment:			\$10,000				
Expected Growt	h Rate:	8.00%	6	Med	. Account Val	ue at Ret:		\$19,671				
Arithmetic Mean	Rate:	9.13%	6	Assı	umed Annuity	Factor:		\$17.93				
Standard Deviat	ion:	15.00	%	Med	ian Initial Ann	ual Income:		\$1,097				
Retirement Ratc	het Assumptior	n: 4%	4% Median Guaranteed Base:									
Scenario Results												
Percent of Simulations Where Income Was Under:												
Survival Probability	Annuity Type	10%	10%   25%   50%   75%   90%   Pr     [Loss]   [Loss]<									
	GIVA at 65	\$835	\$83	5	\$1,097	\$1,523	\$2,029		0.00%			
<sub>10</sub> <b>P</b> <sub>55</sub> = 96.22%	IVA at 65	\$595	\$79	7	\$1,097	\$1,523	\$2	,029	28.20%*			
	GIVA at 70	\$835	\$83	5	\$1,331	\$1,979	\$2	,843	0.00%			
<sub>15</sub> <b>P</b> <sub>55</sub> = 92.62%	IVA at 70	\$622	\$89	8	\$1,329	\$1,978	\$2	,842	23.67%			
	GIVA at 75	\$835	\$94	9	\$1,594	\$2,520	\$3	,814	0.00%			
<sub>20</sub> <b>P</b> <sub>55</sub> = 86.92%	IVA at 75	\$671	\$1,0 <sup>-</sup>	12	\$1,595	\$2,520	\$3	,814	19.13%			
	GIVA at 80	\$835	\$1,1 <sup>-</sup>	13	\$1,924	\$3,223	\$5	,157	0.00%			
<sub>25</sub> <b>P</b> <sub>55</sub> =77.52%	IVA at 80	\$731	\$1,1	54	\$1,925	\$3,224	\$5	,157	15.38%			
	GIVA at 85	\$835	\$1,29	94	\$2,314	\$4,020	\$6	,866	0.00%			
<sub>30</sub> <b>P</b> <sub>55</sub> = 62.83%	IVA at 85	\$800	\$1,32	25	\$2,319	\$4,022	\$6	,866	12.18%			
	GIVA at 90	\$835	\$1,49	91	\$2,772	\$5,076	\$8	,788	0.00%			
<sub>35</sub> <b>P</b> <sub>55</sub> = 42.77%	IVA at 90	\$901	\$1,52	26	\$2,776	\$5,076	\$8	,788	9.88%			
	GIVA at 95	\$835	\$1,74	40	\$3,377	\$6,370	\$1 <sup>-</sup>	1,313	0.00%			
40 <b>P</b> 55 = 22.10%	40P55 = 22.10% IVA at 95 \$1,006 \$1,760 \$3,380 \$6,370 \$11,313 7.99%											
*Analytic confirmation	ation: Pr [IVA <sub>65</sub> <0	GIVA <sub>65</sub> y]=28.70	)%									

Simulation 16: Scenario Parameters											
System A.I.R.:			3%		Initial Investment Age ( <i>y</i> ):				35		
System AM Fee:			0.50%		Anr	Annuitization Age:			65		
System M&E Fee:			0.80%		Initi	Initial Investment:			\$10,000		
Expected Grow	th Rate:		8.00%		Med. Account Value at Ret:				\$76,482		
Arithmetic Mea	n Rate:		9.13%		Ass	Assumed Annuity Factor:				\$17.93	
Standard Devia	ition:		15.00%		Median Initial Annual Income:				\$4,266		
<b>Retirement Rat</b>	chet Assump	tion:	6% Median Guaranteed Base:			\$4,589					
Scenario Results											
	Perce	ent of Simu	ulatio	ons Where Inco	ome Was Und	der:					
Survival Probability	Annuity Type	10%		25%		50%	75%	90%		Pr [Loss]	
	GIVA at 65	\$2,464		\$2,589		\$4,589	\$7,976	\$13,136		0.00%	
<sub>30</sub> <b>P</b> <sub>35</sub> = 94.19%	IVA at 65	\$1,473		\$2,407		\$4,266	\$7,415	\$12,212		100.00%	
	GIVA at 70	\$2,464		\$2,984		\$5,439	\$9,828	\$16,209		0.00%	
<sub>35</sub> <b>P</b> <sub>35</sub> = 90.66%	IVA at 70	\$1,607		\$2,779	)	\$5,095	\$9,338	\$15	910	45.87%	
	GIVA at 75	\$2,464		\$3,371		\$6,374	\$11,887	\$20	951	0.00%	
40 <b>P</b> 35 = 85.09%	IVA at 75	\$1,810		\$3,225		\$6,163	\$11,666	\$20	583	33.72%	
	GIVA at 80	\$2,464		\$3,862		\$7,599	\$14,722	\$26	854	0.00%	
<sub>45</sub> <b>P</b> <sub>35</sub> = 75.88%	IVA at 80	\$2,009		\$3,715		\$7,399	\$14,680	\$26	682	26.86%	
	GIVA at 85	\$2	,464	\$4,451		\$9,032	\$18,308	\$34	571	0.00%	
<sub>50</sub> <b>P</b> <sub>35</sub> = 61.50%	IVA at 85	\$2	,270	\$4,323	3	\$8,908	\$18,308	\$34	571	21.05%	
	GIVA at 90	\$2	,464	\$5,129	)	\$10,858	\$23,004	\$44	973	0.00%	
<sub>55</sub> <b>P</b> <sub>35</sub> = 41.87%	IVA at 90	\$2,563		\$5,040	)	\$10,805	\$23,107	\$44	997	16.89%	
	GIVA at 95	\$2,464		\$5,942		\$12,886	\$28,415	\$58	161	0.00%	
<sub>60</sub> <b>P</b> <sub>35</sub> = 21.64%	IVA at 95	\$2,904		\$5,882		\$12,905	\$28,462	\$58	250	13.83%	
							-				

Simulation 17: Scenario Parameters										
System A.I.R.:		3%	I	Initial Investment Age ( <i>y</i> ):				55		
System AM Fee	0.50%	6	Annuitization Age:			65				
System M&E Fe	e:	0.80%	ω Ι	nitia	I Investment:			\$10,000		
Expected Growt	h Rate:	8.00%	<mark>⁄~</mark>	Med. Account Value at Ret:			\$19,873			
Arithmetic Mean	9.13%	6	Assumed Annuity Factor:				\$17.93			
Standard Deviat	ion:	15.00	15.00%		Median Initial Annual Income:				\$1,108	
Retirement Rato	het Assumption	ı: 6%	6% Median		ian Guarantee	an Guaranteed Base:			\$1,192	
		S	cenario	Res	sults					
		Perce	nt of Simu	ulati	ons Where In	come Was Ur	der:			
Survival Probability	Annuity Type	10%	25%		50%	75%	90%		Pr [Loss]	
	GIVA at 65	\$835	\$860		\$1,192	\$1,636	\$2	,170	0.00%	
<sub>10</sub> <b>P</b> <sub>55</sub> = 96.22%	IVA at 65	\$600	\$800		\$1,108	\$1,520	\$2	,017	100.00%	
	GIVA at 70	\$835	\$971		\$1,405	\$2,022	\$2	,843	0.00%	
<sub>15</sub> <b>P</b> <sub>55</sub> = 92.62%	IVA at 70	\$631	\$906		\$1,341	\$1,970	\$2	,835	42.54%	
	GIVA at 75	\$835	\$1,086		\$1,648	\$2,563	\$3	,782	0.00%	
<sub>20</sub> <b>P</b> <sub>55</sub> = 86.92%	IVA at 75	\$692	\$1,024		\$1,611	\$2,571	\$3	,793	30.36%	
	GIVA at 80	\$835	\$1,224	1	\$1,950	\$3,232	\$5	,147	0.00%	
<sub>25</sub> <b>P</b> <sub>55</sub> =77.52%	IVA at 80	\$751	\$1,167		\$1,934	\$3,255	\$5	,162	23.55%	
	GIVA at 85	\$835	\$1,369	)	\$2,330	\$4,108	\$6	,871	0.00%	
<sub>30</sub> <b>P</b> <sub>55</sub> = 62.83%	IVA at 85	\$815	\$1,353	3	\$2,345	\$4,119	\$6	,875	19.08%	
	GIVA at 90	\$923	\$1,573	3	\$2,817	\$5,222	\$9	,030	0.00%	
<sub>35</sub> <b>P</b> <sub>55</sub> = 42.77%	IVA at 90	\$902	\$1,562	2	\$2,843	\$5,223	\$9	,030	15.12%	
	GIVA at 95	\$1,010	\$1,754		\$3,403	\$6,568	\$1 <sup>·</sup>	1,688	0.00%	
40 <b>P</b> 55 = 22.10%	IVA at 95	\$1,013	\$1,772	2	\$3,441	\$6,579	\$1 <sup>·</sup>	1,688	12.42%	

Simulation 18: Option's Worth Scenario Parameters							
System A.I.R.:	3%	Initial Investment Age ( <i>y</i> ):	65				
System AM Fee:	0.00%	Annuitization Age:	65				
System M&E Fee:	0.00%	Initial Investment:	\$10,000				
Expected Growth Rate:	1.90%	Med. Account Value at Ret:	\$10,000				
Arithmetic Mean Rate:	3.0%	Annuity Factor:	\$17.93				
Standard Deviation:	15.00%	Median Initial Annual Income:	\$558				
<b>Retirement Ratchet Assumption:</b>	5%	Median Guaranteed Base:	\$500				
Simulation 19: Option's Worth Scenario Parameters							
System A.I.R.:	3%	Initial Investment Age ( <i>y</i> ):	35				
System AM Fee:	0.00%	Annuitization Age:	65				
System M&E Fee:	0.80%	Initial Investment:	\$10,000				
Expected Growth Rate:	1.90%	Med. Account Value at Ret:	\$13,822				
Arithmetic Mean Rate:	3.0%	Assumed Annuity Factor:	\$17.93				
Standard Deviation:	15.00%	Median Initial Annual Income:	\$771				
<b>Retirement Ratchet Assumption:</b>	5%	Median Guaranteed Base:	\$2,464				
Simulation 20: Option's Worth Scenario Parameters							
System A.I.R.:	3%	Initial Investment Age ( <i>y</i> ):	45				
System AM Fee:	0.00%	Annuitization Age:	65				
System M&E Fee:	0.80%	Initial Investment:	\$10,000				
Expected Growth Rate:	1.90%	Med. Account Value at Ret:	\$12,433				
Arithmetic Mean Rate:	3.0%	Assumed Annuity Factor:	\$17.93				
Standard Deviation:	15.00%	Median Initial Annual Income:	\$694				
<b>Retirement Ratchet Assumption:</b>	5%	Median Guaranteed Base:	\$1,554				
Simulation 21: Option's Worth Scenario Parameters							
System A.I.R.:	3%	Initial Investment Age ( <i>y</i> ):	55				
System AM Fee:	0.00%	Annuitization Age:	65				
System M&E Fee:	0.80%	Initial Investment:	\$10,000				
Expected Growth Rate:	1.90%	Med. Account Value at Ret:	\$11,213				
Arithmetic Mean Rate:	3.0%	Assumed Annuity Factor:	\$17.93				
Standard Deviation:	15.00%	Median Initial Annual Income:	\$625				
<b>Retirement Ratchet Assumption:</b>	5%	Median Guaranteed Base:	\$835				
Simulation 22: Option's Worth Scenario Parameters							
System A.I.R.:	3%	Initial Investment Age (y):	65				
System AM Fee:	0.00%	Annuitization Age:	65				
System M&E Fee:	0.80%	Initial Investment:	\$10,000				
Expected Growth Rate:	1.90%	Med. Account Value at Ret:	\$10,000				
Arithmetic Mean Rate:	3.0%	Annuity Factor:	\$17.93				
Standard Deviation:	15.00%	Median Initial Annual Income:	\$558				
<b>Retirement Ratchet Assumption:</b>	5%	Median Guaranteed Base:	\$500				

Simulation 23: Option's Worth Scenario Parameters						
System A.I.R.:	3%	Initial Investment Age ( <i>y</i> ):	35			
System AM Fee:	0.00%	Annuitization Age:	65			
System M&E Fee:	0.80%	Initial Investment:	\$10,000			
Expected Growth Rate:	1.90%	Med. Account Value at Ret:	\$13,781			
Arithmetic Mean Rate:	3.0%	Assumed Annuity Factor:	\$20.75			
Standard Deviation:	15.00%	Median Initial Annual Income:	\$664			
Retirement Ratchet Assumption:	5%	Median Guaranteed Base:	\$2,464			
Simulation 24: Option's Worth Scenario Parameters						
System A.I.R.:	3%	Initial Investment Age (y):	45			
System AM Fee:	0.00%	Annuitization Age:	65			
System M&E Fee:	0.80%	Initial Investment:	\$10,000			
Expected Growth Rate:	1.90%	Med. Account Value at Ret:	\$12,644			
Arithmetic Mean Rate:	3.0%	Assumed Annuity Factor:	\$19.77			
Standard Deviation:	15.00%	Median Initial Annual Income:	\$640			
Retirement Ratchet Assumption:	5%	Median Guaranteed Base:	\$1,554			
Simulation 25: Option's Worth Scenario Parameters						
System A.I.R.:	3%	Initial Investment Age (y):	55			
System AM Fee:	0.00%	Annuitization Age:	65			
System M&E Fee:	0.80%	Initial Investment:	\$10,000			
Expected Growth Rate:	1.90%	Med. Account Value at Ret:	\$11,095			
Arithmetic Mean Rate:	3.0%	Assumed Annuity Factor:	\$18.82			
Standard Deviation:	15.00%	Median Initial Annual Income:	\$589			
Retirement Ratchet Assumption:	5%	Median Guaranteed Base:	\$835			

The following two tables illustrate the results of our simulation analysis for the Option's Worth of the embedded guarantee. Results for Scenarios 18-22 are based on the assumption that the insurance company will continue to use the IAM2000 (female) mortality table when annuitizing the account at retirement. Results for Scenarios 23-25 are based on the assumption that every ten years the insurance company will (arbitrarily) increase the annuity factor ( $a_{65} = 17.93$ ) by 5% to account for increasing mortality.

For example, a 35 year-old investing \$10,000 in the GIVA would be receiving a benefit worth \$4,145 (which is 41% of the premium deposit), if the current IAM2000 table continues to be used for the next 30 years. However, if the mortality table factors are updated, then the benefit is slightly reduced to \$3,897 (or 39% of the premium deposit).

Scenarios 18-22: Results						
\$10,000 invested in a Guaranteed VPA: Option's Worth – Current Mortality Table						
Retirement Annuitization Age: 65						
Simulation #	Investment Age	Expected Benefits Minus Fees	Option's Worth			
18	65	\$11,153	12%			
19	35	\$14,145	41%			
20	45	\$12,664	27%			
21	55	\$10,403	4%			
22	65	\$9,673	-3%			

Scenarios 23-25: Results						
\$10,000 invested in a Guaranteed VPA: Option's Worth – with Dynamic Projection of Mortality						
Retirement Annuitization Age: 65						
Simulation #	Investment Age	Expected Benefits Minus Fees	Option's Worth			
18	65	\$11,153	12%			
23	35	\$13,897	39%			
24	45	\$12,512	25%			
25	55	\$10,216	2%			
22	65	\$9,673	-3%			